

CLASS - X

TELANGANA



MODEL PAPER

2

MATHEMATICS : PAPER - II

MARCH 2019

Time : 2.45 Hours]

Parts - A and B

[Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle,
 Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. Read the whole question paper and understand every question thoroughly without writing anything and 15 minutes of time is allotted for this.
2. Answer the questions under **Part - A** on a separate answer book.
3. Write the answers to the questions under **Part - B** on the question paper itself and attach it to the answer book of **Part - A**.
4. Answer **all** the questions from the given Three sections I, II and III of **Part - A**.
5. In section III, every question has internal choice. Answer any **one** alternative.

Time : 2.00 Hours]

PART - A

[Marks : 35

SECTION - I

(Marks : $7 \times 1 = 7$)Note : (i) Answer **all** the following questions.

(ii) Each question carries 1 mark.

1. Evaluate $\operatorname{cosec} 39^\circ \cdot \sec 51^\circ - \tan 51^\circ \cdot \cot 39^\circ$.
2. Write the similarity criterion by which the given pair of triangles are similar.
3. From English alphabet if a letter is chosen at random, then find the probability that the letter is a consonant.
4. In a right triangle ABC, right angled at C in which $AB = 13$ cm, $BC = 5$ cm, determine the value of $\cos^2 B + \sin^2 A$.
5. A point P is 25 cm from the centre O of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.
6. Find the median of first seven composite numbers.
7. In a hemispherical bowl of 2.1 cm radius ice-cream is there. Find the volume of the bowl.

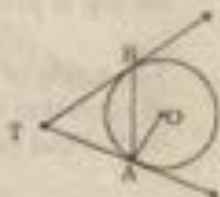


SECTION - II

(Marks : $6 \times 2 = 12$)Note : (i) Answer **all** the questions.

(ii) Each question carries 2 marks.

8. Write the mode formula for grouped data and explain the terms in it.
9. In the given figure, TA and TB are tangents to the circle with centre O. If $\angle ATB = 80^\circ$, then find the measure of $\angle ABT$.



10. A bag contains balls which are numbered from 1 to 50. A ball is drawn at random from the bag, the probability that it bears a two digit number multiple of 7.
11. From the top of the building the angle of elevation of the top of the cell tower is 60° and the angle of depression to its foot is 45° , if the distance of the building from the tower is 30 meters, draw the suitable diagram to the given data.
12. Find the value of $\frac{\tan^2 60^\circ + \cot^2 30^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$.
13. A right circular cylinder has radius 3.5 cm and height 14 cm. Find curved surface area.

SECTION - III

(Marks : $4 \times 4 = 16$)

Note : (i) Answer **all** the following questions.

(ii) In this section, every question has internal choice to answer.

(iii) Each question carries **4** marks.

14. Construct a triangle PQR, in which $PQ = 4$ cm, $QR = 6$ cm and $\angle PQR = 70^\circ$. Construct triangle such that each side of the new triangle is $\frac{3}{4}$ of the triangle of PQR.

OR

Draw less than Ogive for the following frequency distribution. Find the median from obtained curve.

IQ	60 - 70	70 - 80	80 - 90	90 - 100	100 - 110	110 - 120	120 - 130
No. of students	2	5	12	31	39	10	4

15. Show that $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$.

OR

In a right angle triangle, the hypotenuse is 10 cm more than the shortest side. If third side is 6 cm less than the hypotenuse, find the sides of the right angle triangle.

16. Find the mean age of 100 residents of a colony from the following data.

Age (in years)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Persons	10	15	25	25	10	10	5

OR

A toy is made with seven equal cubes of sides $\sqrt{7}$ cm. Six cubes are joined to six faces of a seventh cube. Find the total surface area of the toy.

17. If two dice are thrown at the same time, find the probability of getting sum of the dots on top is prime.

OR

The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high. Find the height of the hill.

Note :

- (i) Answer all the questions.
 (ii) Each question carries $\frac{1}{2}$ mark.
 (iii) Answers are to be written in question paper only.
 (iv) Marks will **not** be awarded in any case of overwriting, rewriting or erased answers.
 (v) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

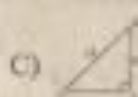
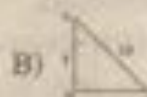
1. Number of secants that can be drawn to circle through a point inside it is []
 A) 0 B) 1 C) Infinite D) 2
2. If the less than type Ogive and more than type Ogive intersect each other at (42, 18), then the median of the given data is []
 A) 60 B) 42 C) 18 D) 26
3. A 20 m long ladder is placed on a pole of 10 m height making ' α ' angle with the ground, then $\alpha =$ []
 A) 60° B) 45° C) 30° D) 0°
4. Tangents PA and PB inclined at an angle 60° as shown in the figure, the ratio of lengths of OA, OP and AP is []
 A) 1 : 2 : 3 B) 3 : 2 : 1
 C) $\sqrt{3} : 2 : 1$ D) 1 : 2 : $\sqrt{3}$
5. Ratio of areas of two similar triangles is 144 : 441, then ratio of their perimeters is []
 A) 14 : 41 B) 44 : 41 C) 7 : 4 D) 4 : 7
6. A cylinder and a cone have equal radii and equal heights. If the volume of cylinder is 27 cu. units, then the volume of cone is []
 A) 27 cu. units B) 18 cu. units C) 9 cu. units D) 36 cu. units



7. Area of the shaded region is []
 A) $r^2(2 - \pi)$ B) $r^2(4 - \pi)$
 C) $r^2(5 - \pi)$ D) $r^2(6 - \pi)$



8. Choose the correct figure for which $\sin A = \frac{5}{13}$ []



9. A letter is chosen from the word "BAHUBALI", the probability that it was not a vowel is []

A) $\frac{1}{2}$

B) $\frac{3}{2}$

C) $\frac{4}{3}$

D) $\frac{3}{4}$

10. Which of the following statement is true ? []

- A) All acute angle triangles are similar. B) All obtuse angle triangles are similar.
 C) All right angle triangles are similar. D) All isosceles right triangles are similar.

SOLUTIONS

PART - A

SECTION - I

1. Evaluate $\operatorname{cosec} 39^\circ \cdot \sec 51^\circ - \tan 51^\circ \cdot \cot 39^\circ$.

Sol. $\sec A = \operatorname{Cosec} (90^\circ - A)$

$$\sec 51^\circ = \operatorname{Cosec} (90^\circ - 51^\circ) = \operatorname{Cosec} 39^\circ$$

$$\tan A = \cot (90^\circ - A)$$

$$\tan 51^\circ = \cot (90^\circ - 51^\circ) = \cot 39^\circ$$

$$\begin{aligned} \operatorname{Cosec} 39^\circ \cdot \sec 51^\circ - \tan 51^\circ \cdot \cot 39^\circ \\ = \operatorname{Cosec} 39^\circ \cdot \operatorname{Cosec} 39^\circ - \cot 39^\circ \cdot \cot 39^\circ \\ = \operatorname{Cosec}^2 39^\circ - \cot^2 39^\circ \end{aligned}$$

$$(\therefore \operatorname{Cosec}^2 A - \cot^2 A = 1)$$

$$= 1$$

2. Write the similarity criterion by which the given pair of triangles are similar.



Sol. $\frac{OA}{OB} = \frac{3}{6} = \frac{1}{2}$

$$\frac{OC}{OD} = \frac{2.5}{5} = 0.5 = \frac{1}{2}$$

$$\frac{OA}{OB} = \frac{OC}{OD} \Rightarrow \angle AOC = \angle BOD$$

Using SAS - Criterion $\therefore \triangle OAC \sim \triangle OBD$

3. From English alphabet if a letter is chosen at random, then find the probability that the letter is a consonant.

Sol. Number of total outcomes = 26

Number of favourable outcomes = 21

Probability that the letter is consonant

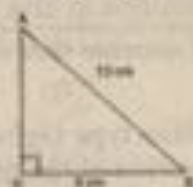
$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$= \frac{21}{26}$$

4. In a right triangle ABC, right angled at 'C' in which AB = 13 cm, BC = 5 cm, determine the value of $\cos^2 B + \sin^2 A$.

Sol. We have, $\cos B = \frac{BC}{AB} = \frac{5}{13}$

$$\sin A = \frac{BC}{AB} = \frac{5}{13}$$



$$\cos^2 B + \sin^2 A$$

$$= \frac{25}{169} + \frac{25}{169} = \frac{25+25}{169} = \frac{50}{169}$$

5. A point P is 25 cm from the centre O of the circle. The length of the tangent drawn from P to the circle is 24 cm. Find the radius of the circle.

Sol. From right angled $\triangle AOP$



$$OP^2 = OA^2 + AP^2$$

$$(25)^2 = OA^2 + (24)^2$$

$$625 = OA^2 + 576$$

$$OA^2 = 625 - 576 = 49 = 7^2$$

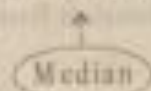
$$OA = 7 \text{ cm}$$

\therefore The radius of the circle is 7 cm.

6. Find the median of first seven composite numbers.

Sol. The first seven composite numbers are

4, 6, 8, (9), 10, 12, 14



\therefore Median = 9

7. In a hemispherical bowl of 2.1 cm radius ice-cream is there. Find the volume of the bowl.

Sol. Radius (r) = 2.1 cm

$$\begin{aligned} \text{Volume (V)} &= \frac{2}{3} \pi r^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times (2.1)^3 \\ &= \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1^{(2.1^3)} \\ &= 19.404 \text{ cm}^3 \end{aligned}$$

SECTION - II

8. Write the mode formula for grouped data and explain the terms in it.

Sol. Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right] \times h$

l = lower boundary of mode class

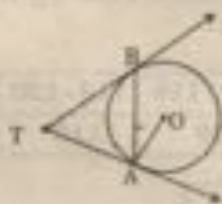
f_1 = frequency of mode class

f_0 = frequency of preceding class to the mode class

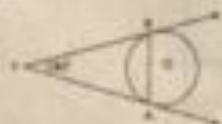
f_2 = frequency of succeeding class to the mode class

h = size of class

9. In the given figure, TA and TB are tangents to the circle with centre 'O'. If $\angle ATB = 80^\circ$, then find the measure of $\angle ABT$.



Sol. In $\triangle TAB$,



TA = TB (\because Length of the tangents drawn from the external point are equal)

$$\Rightarrow \angle TBA = \angle TAB$$

(\because Angles opposite to the equal sides are equal)

$$\Rightarrow \text{Let } \angle TBA = \angle TAB = x^\circ$$

$$80^\circ + x^\circ + x^\circ = 180^\circ \quad (\because \text{Sum of the angles in a triangle is } 180^\circ)$$

$$2x^\circ = 180^\circ - 80^\circ = 100^\circ$$

$$x^\circ = 50^\circ$$

$$\therefore \angle ABT = 50^\circ$$

10. A bag contains balls which are numbered from 1 to 50. A ball is drawn at random from the bag, the probability that it bears a two digit number multiple of 7.

Sol. Number of possible outcomes = 50

Number of required outcomes = 6

{14, 21, 28, 35, 42, 49}

\therefore Probability of getting two digit number which is a multiple of 7

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$= \frac{6}{50} = \frac{3}{25}$$

11. From the top of the building the angle of elevation of the top of the cell tower is 60° and the angle of depression to its foot is 45° , if the distance of the building from the tower is 30 meters, draw the suitable diagram to the given data.

Sol. AB - Height of the building

CD - Height of the tower

$\angle CAE$ - angle of elevation = 60°

$\angle EAD$ - angle of depression = 45°

Distance between building and tower = BD = 30m



12. Find the value of $\frac{\tan^2 60^\circ + \cot^2 30^\circ}{\sin^2 30^\circ + \cos^2 60^\circ}$.

Sol.
$$\frac{\tan^2 60^\circ + \cot^2 30^\circ}{\sin^2 30^\circ + \cos^2 60^\circ} = \frac{(\sqrt{3})^2 + (\frac{1}{\sqrt{3}})^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$= \frac{3 + \frac{1}{3}}{\frac{1}{4} + \frac{1}{4}} = \frac{\frac{10}{3}}{\frac{1}{2}} = \frac{10}{3} \times \frac{2}{1} = \frac{20}{3}$$

13. A right circular cylinder has radius 3.5 cm and height 14 cm. Find curved surface area.

Sol. radius (r) = 3.5 cm

height (h) = 14 cm

Curved surface area of the right circular cylinder

$$= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 14$$

$$= 44 \times 7 = 308 \text{ cm}^2$$

SECTION - III

14. Construct a triangle PQR, in which PQ = 4 cm, QR = 6 cm and $\angle PQR = 70^\circ$. Construct triangle such that each side of the new triangle is $\frac{3}{4}$ of the triangle of PQR.

Sol. Steps of construction :

1. Draw a triangle PQR with given measurements.



2. Draw \overline{PY} such that $\angle QPY$ is an acute angle.
3. Locate points A_1, A_2, A_3, A_4 on \overline{PY} such that $PA_1 = A_1A_2 = A_2A_3 = A_3A_4$.
4. Join A_4 and Q.
5. Draw a parallel line to A_4Q through A_3 to meet PQ at Q^1 .
6. Draw a parallel line to QR through Q^1 to meet PR at R^1 .
7. $\triangle PQ^1R^1$ is required similar triangle.

OR

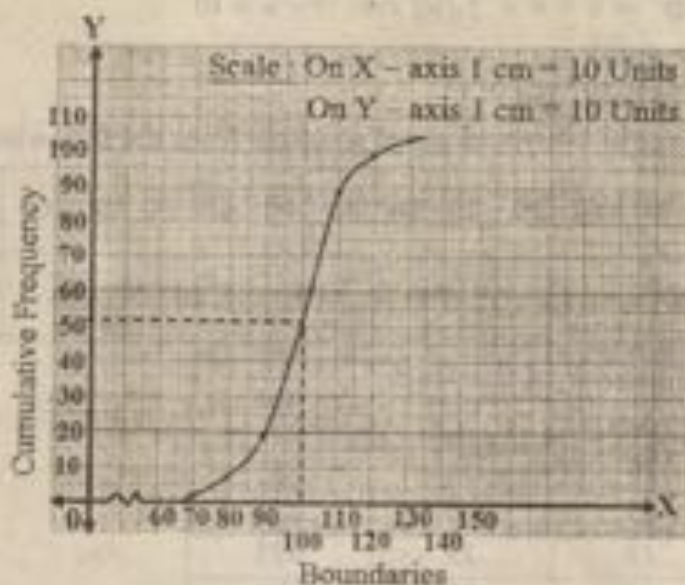
Draw less than Ogive for the following frequency distribution. Find the median from obtained curve.

IQ	60 - 70	70 - 80	80 - 90	90 - 100	100 - 110	110 - 120	120 - 130
No. of students	2	5	12	31	39	10	4

Sol.

IQ (in years)	No. of Students	Upper bound frequency	Less than cumulative	(x, y)
60 - 70	2	70	2	(70, 2)
70 - 80	5	80	7	(80, 7)
80 - 90	12	90	19	(90, 19)
90 - 100	31	100	50	(100, 50)
100 - 110	39	110	89	(110, 89)
110 - 120	10	120	99	(120, 99)
120 - 130	4	130	103	(130, 103)

Here $n = 103$



$$n = 103 \Rightarrow \frac{n}{2} = \frac{103}{2} = 51.5$$

From graph, median = 100

15. Show that $\frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} = 2 \sec \theta$.

Sol. LHS :

$$\begin{aligned} \frac{\cos \theta}{1 - \sin \theta} + \frac{1 - \sin \theta}{\cos \theta} &= \frac{\cos^2 \theta + (1 - \sin \theta)^2}{(1 - \sin \theta) \cos \theta} \\ &= \frac{\cos^2 \theta + 1 - 2 \sin \theta + \sin^2 \theta}{(1 - \sin \theta) \cos \theta} \\ &= \frac{\cos^2 \theta + \sin^2 \theta + 1 - 2 \sin \theta}{(1 - \sin \theta) \cos \theta} \\ &= \frac{1 + 1 - 2 \sin \theta}{(1 - \sin \theta) \cos \theta} = \frac{2 - 2 \sin \theta}{(1 - \sin \theta) \cos \theta} \\ &= \frac{2(1 - \sin \theta)}{(1 - \sin \theta) \cos \theta} \\ &= 2 \cdot \frac{1}{\cos \theta} = 2 \sec \theta = \text{RHS} \end{aligned}$$

LHS = RHS

OR

In a right angle triangle, the hypotenuse is 10 cm more than the shortest side. If third side is 6 cm less than the hypotenuse, find the sides of the right angle triangle.

Sol. Let the shortest side of right angled triangle be x cm

Hypotenuse = $(x + 10)$ cm (given)

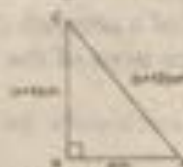
Third side = $(x + 10 - 6)$ cm

= $(x + 4)$ cm (given)

AC = $(x + 10)$ cm, AB = x cm,

BC = $(x + 4)$ cm.

In a right angled triangle ABC



$$AC^2 = AB^2 + BC^2$$

$$(x + 10)^2 = x^2 + (x + 4)^2$$

$$x^2 + 20x + 100 = x^2 + x^2 + 8x + 16$$

$$\Rightarrow x^2 - 12x - 84 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-12)^2 - 4.1(-84)}}{2 \times 1}$$

$$= \frac{12 \pm \sqrt{144 + 336}}{2} = \frac{12 \pm \sqrt{480}}{2}$$

$$= \frac{12 \pm \sqrt{16 \times 30}}{2} = \frac{12 \pm 4\sqrt{30}}{2}$$

$$x = 6 \pm 2\sqrt{30} \Rightarrow x = 6 + 2\sqrt{30} \text{ cm } (\because x > 0)$$

$$x + 4 = 10 + 2\sqrt{30} \text{ cm}$$

$$x + 10 = 16 + 2\sqrt{30} \text{ cm}$$

16. Find the mean age of 100 residents of a colony from the following data.

Age (in years)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Persons	10	15	25	25	10	10	5

Sol.

Age (in years)	Number of Persons (f)	Class Mark x	fx
0 - 10	10	5	50
10 - 20	15	15	225
20 - 30	25	25	625
30 - 40	25	35	875
40 - 50	10	45	450
50 - 60	10	55	550
60 - 70	5	65	325

Here $\Sigma f = 100$ $\Sigma fx = 3100$

$$\text{Mean} = \bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{3100}{100} \Rightarrow \therefore \text{Mean} = 31$$

OR

A toy is made with seven equal cubes of sides $\sqrt{7}$ cm. Six cubes are joined to six faces of a seventh cube. Find the total surface area of the toy.

Sol. Let side of each cube be $a = \sqrt{7}$ cm

Total surface area of Seven Cubes

$$= 7 \times 6a^2 = 42a^2$$

Total surface area of common portion in the toy = $12a^2$

\therefore Total surface area of a Toy

$$= 42a^2 - 12a^2$$

$$= 30a^2$$

$$= 30 \times (\sqrt{7})^2$$

$$= 30 \times 7 = 210 \text{ cm}^2$$

17. If two dice are thrown at the same time, find the probability of getting sum of the dots on top is prime.

Sol. Total possible outcomes when two dice are rolled = 36

Let E be an event to get sum of the tops is prime

Total favourable outcomes :

(1, 1) (1, 2), (2, 1), (1, 4), (2, 3), (3, 2), (4, 1), (1, 6), (2, 5), (3, 4), (4, 3) (5, 2), (6, 1), (5, 6), (6, 5)

Number of total favourable outcomes = 15

$$P(E) = \frac{\text{No. of favourable outcomes}}{\text{No. of total possible outcomes}}$$

$$= \frac{15}{36} = \frac{5}{12}$$

OR

The angle of elevation of the top of a hill from the foot of a tower is 60° and the angle of elevation of the top of the tower from the foot of the hill is 30° . If the tower is 50 m high. Find the height of the hill.

Given height of the tower = $AB = 50$ m

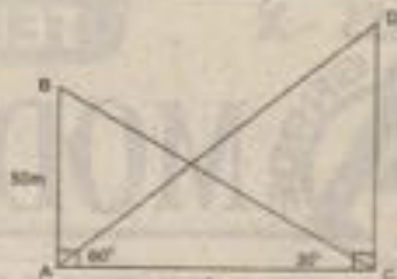
Let height of hill be $CD = h$ m

and distance between their feet be AC

= x m

$\angle ACB = 30^\circ$, $\angle CAD = 60^\circ$

From right angled $\triangle ABC$, $\tan 30^\circ = \frac{AB}{AC}$



$$\frac{1}{3} = \frac{50}{x} \Rightarrow x = 50\sqrt{3} \text{ m.}$$

From right angled $\triangle ACD$, $\tan 60^\circ = \frac{CD}{AC}$

$$\sqrt{3} = \frac{h}{x} \Rightarrow h = x\sqrt{3} \text{ m}$$

$$h = 50\sqrt{3} \cdot \sqrt{3} \quad (\because x = 50\sqrt{3} \text{ m})$$

$$h = 50 \times 3 \Rightarrow h = 150 \text{ m.}$$

PART - B

- 1) C 2) B 3) C 4) D(Add Score) 5) D 6) C 7) B 8) D
9) A 10) D

