

CLASS - X

TELANGANA



MODEL PAPER

4

MATHEMATICS : PAPER - II

MARCH 2018

Time : 2.45 Hours]

Parts - A and B

[Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle,
Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. Read the whole question paper and understand every question thoroughly without writing anything and 15 minutes of time is allotted for this.
2. Answer the questions under **Part - A** on a separate answer book.
3. Write the answers to the questions under **Part - B** on the question paper itself and attach it to the answer book of **Part - A**.
4. Answer **all** the questions from the given Three sections I, II and III of **Part - A**.
5. In section III, every question has internal choice, Answer any **one** alternative.

Time : 2.00 Hours]

PART - A

[Marks : 35

SECTION - I

[Marks : $7 \times 1 = 7$]

Note : i) Answer **all** the following questions.
ii) Each question carries **1** Mark.

1. Prathyusha stated that "the average of first 10 odd numbers is also 10". Do you agree with her? Justify your answer.
2. Write the formula to find the median of a grouped data and explain the alphabet in it.
3. The length of the tangent to a circle from a point 17 cm from its centre is 8 cm. Find the radius of the circle.
4. Find the value of $\tan 2A$, if $\cos 3A = \sin 45^\circ$.
5. Srivani walks 12 m due East and turns left and walks another 5 m, how far is she from the place she started?
6. A pole and its shadow have same length, find the angle of the sun ray made with the earth at that time.
7. What is the probability of getting exactly two heads, when three coins tossed simultaneously?

SECTION - II**(Marks : 6 × 2 = 12)****Note :** i) Answer all the questions.

ii) Each question carries 2 Marks.

8. Find measure of the angles A and B, if $\cos(A - B) = \frac{\sqrt{3}}{2}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$.
9. What is the probability of a number picked from first twenty natural numbers is even composite number ?
10. From the top of a tower of h m height, Anusha observes the angles of depression of two points X and Y on the same side of tower on the ground to be α and β . Draw the suitable figure for the given information.
11. Find the median of $\frac{2}{3}, \frac{4}{5}, \frac{1}{2}, \frac{3}{4}, \frac{6}{5}$
12. The height and the base radius of a Cone and a Cylinder are equal to the radius of a Sphere. Find the ratio of their volumes.
13. In $\triangle ABC$, $\overline{PQ} \parallel \overline{BC}$ and $AP = 3x - 19$, $PB = x - 5$, $AQ = x - 3$, $QC = 3$ cm. Find x.

SECTION - III**(Marks : 4 × 4 = 16)****Note :** i) Answer all the following questions..

ii) In this section, every question has internal choice to answer.

iii) Each question carries 4 marks.

14. How many silver coins of diameter 5 cm and thickness 4 mm have to be melted to prepare a cuboid of 12 cm × 11 cm × 5 cm dimension ?

OR

Incomes of the families in a locality are given. Find the mode of the data.

Income (in ₹)	1 - 200	201 - 400	401 - 600	601 - 800	801 - 1000
Number of families	7	10	16	12	3

15. Prove that : $\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$.

ORShow that : $(\sec \theta - \tan \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$.

16. From the top of a tower of 50 m high, Neha observes the angles of depression of the top and foot of another building to be 45° and 60° respectively. Find the height of the building.

OR

From the deck of 52 cards, if a card is randomly chosen, find the probability of getting a card with

(i) a prime number on it, (ii) face on it.

17. Construct an equilateral triangle XYZ of side 5 cm and construct another triangle similar to $\triangle XYZ$, such that each of its sides is $\frac{4}{5}$ of the sides of $\triangle XYZ$.

OR

Heights of the pupils of a particular school are given. Draw greater than cumulative curve and find the median height from it.

Height (in cm)	90-100	100-110	110-120	120-130	130-140	140-150
Number of pupils	5	2	3	8	8	6


Time : 30 Mts.]

PART - B

[Marks : 5

Note :

- Answer **all** the questions.
- Each question carries $\frac{1}{2}$ mark.
- Answers are to be written in question paper only.
- Marks will **not** be awarded in any case of overwriting, rewriting or erased answers.
- Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

- The most stable measure of central tendency is []
 A) Mean B) Median C) Mode D) Deviation
- If $P(E)$ is the probability of an event E , then []
 A) $0 < P(E) < 1$ B) $0 \leq P(E) < 1$ C) $0 \leq P(E) \leq 1$ D) $0 < P(E) \leq 1$
- The perimeters of two similar triangles are in 4 : 9 ratio, the ratio of their corresponding sides is []
 A) 9 : 4 B) 2 : 3 C) 16 : 81 D) 4 : 9
- If $\cot \theta - \operatorname{cosec} \theta = p$, then $\cot \theta + \operatorname{cosec} \theta =$ []
 A) p B) $-p$ C) $\frac{1}{p}$ D) $-\frac{1}{p}$
- If \overline{AP} and \overline{AQ} are two tangents to a circle with centre O , such that $\angle POQ = 105^\circ$, then $\angle PAQ =$ []
 A) 105° B) 90° C) 75° D) 65°

- The volume of a cone with base radius 7 cm is 462 c.c., its height is []
 A) 9 cm B) 18 cm C) 3 cm D) 27 cm
- A ladder touches a wall at a height of 5 m. The angle made by the ladder with the ground, if its length is 10 m, will be []
 A) 30° B) 60° C) 45° D) 90°
- $\tan \theta$ in terms of $\cos \theta$ is []
 A) $\frac{\sqrt{1 + \cos^2 \theta}}{\cos \theta}$ B) $\frac{\cos \theta}{\sqrt{1 + \cos^2 \theta}}$ C) $\frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$ D) $\frac{\cos \theta}{\sqrt{1 - \cos^2 \theta}}$
- The probability of getting right answer to a question is 0.68, the probability of getting a wrong answer is []
 A) 0.32 B) 32% C) 32 D) A and B
- \overline{AB} is a tangent drawn to a circle with centre O from an external point A and B is a point of contact, then which of the following is always true? []
 (i) $OA > OB$ (ii) $OA > AB$ (iii) $AB > OB$
 A) only (i) B) only (ii) C) (ii) and (iii) D) (i) and (ii)

SOLUTIONS

PART - A

SECTION - I

1. Prathyusha stated that "the average of first 10 odd numbers is also 10". Do you agree with her? Justify your answer.

Sol. The average of first 10 odd numbers

$$= \frac{10 [1+19]}{2 \cdot 10} = \frac{5 \times 20}{10} = \frac{100}{10} = 10$$
 \therefore The average of first 10 odd numbers is 10.

I agree with Prathyusha statement.

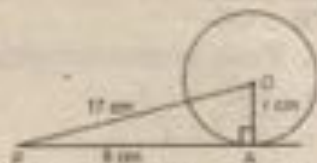
2. Write the formula to find the median of a grouped data and explain the alphabet in it.

Sol. Median = $l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$

l = lower boundary of median class.
 n = number of observations.
 cf = cumulative frequency of class preceding the median class.
 f = frequency of median class.
 h = class size.

3. The length of the tangent to a circle from a point 17 cm from its centre is 8 cm. Find the radius of the circle.

Sol. Let PA is the length of the tangent
 $PA = 8$ cm



The distance of the external point from the centre $OP = 17$ cm

Radius = $OA = r$ cm.

$$\therefore 17^2 = r^2 + 8^2$$

$$\Rightarrow r^2 = 17^2 - 8^2 = 289 - 64 = 225$$

$$\Rightarrow r = \sqrt{225} = 15 \text{ cm}$$

4. Find the value of $\tan 2A$, if

$$\cos 3A = \sin 45^\circ.$$

Sol. $\cos 3A = \sin 45^\circ$

$$\cos 3A = \frac{1}{\sqrt{2}} = \cos 45^\circ$$

$$\Rightarrow 3A = 45^\circ \quad \left(\because \sin 45^\circ = \frac{1}{\sqrt{2}} \right)$$

$$\therefore A = \frac{45^\circ}{3} = 15^\circ$$

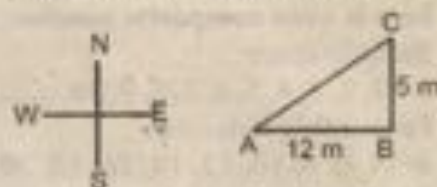
$$\tan 2A = \tan (2 \times 15^\circ) = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

5. Srivani walks 12 m due East and turns left and walks another 5 m. how far is she from the place she started?

Sol. The distance of Srivani from the place she started

$$= \sqrt{12^2 + 5^2}$$

$$= \sqrt{144 + 25} = \sqrt{169} = 13 \text{ m}$$



6. A pole and its shadow have same length, find the angle of the sun ray made with the earth at that time.

Sol. Let the height of the pole
 = the length of its shadow = h

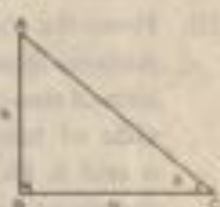
$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$= \frac{AB}{BC}$$

$$= \frac{h}{h} = 1$$

$$\therefore \tan \theta = \tan 45^\circ$$

$$\theta = 45^\circ$$



7. What is the probability of getting exactly two heads, when three coins tossed simultaneously?

Sol. Probability of getting exactly two heads

$$= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{3}{8}$$

SECTION - II

8. Find measure of the angles A and B, if

$$\cos(A - B) = \frac{\sqrt{3}}{2} \text{ and}$$

$$\sin(A + B) = \frac{\sqrt{3}}{2}$$

Sol. $\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30^\circ$
 $A - B = 30^\circ$ ——— (1)

$$\sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60^\circ$$

$$A + B = 60^\circ$$
 ——— (2)

$$(1) + (2) \Rightarrow A + B = 60$$

$$A - B = 30$$

$$2A = 90^\circ$$

$$\therefore A = 45^\circ; B = 15^\circ$$

9. What is the probability of a number picked from first twenty natural numbers is even composite number?

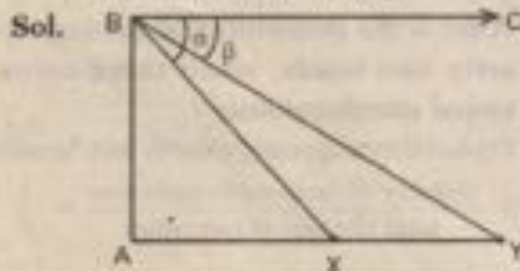
Sol. Sample space
 $= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \dots, 20\}$
 Favourable outcomes
 $= \{4, 6, 8, 10, 12, 14, 16, 18, 20\}$
 Probability of getting even composite number

from first twenty natural numbers

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$= \frac{9}{20}$$

10. From the top of a tower of h m height, Anusha observes the angles of depression of two points X and Y on the same side of tower on the ground to be α and β . Draw the suitable figure for the given information.



11. Find the median of $\frac{2}{3}, \frac{4}{5}, \frac{1}{2}, \frac{3}{4}, \frac{6}{5}$.

Sol. $\frac{1}{2}, \frac{2}{3}, \left(\frac{3}{4}\right), \frac{4}{5}, \frac{6}{5}$

↓
median

$$\therefore \text{Median} = \frac{3}{4}$$

12. The height and the base radius of a Cone and a Cylinder are equal to the radius of a Sphere. Find the ratio of their volumes.

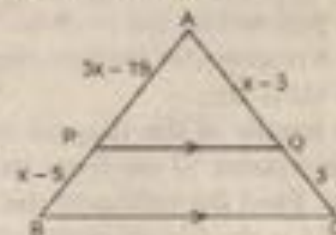
Sol. The ratio of the volumes of the cone, cylinder and spheres

$$= \frac{1}{3} \pi r^3 : \pi r^3 : \frac{4}{3} \pi r^3$$

$$= \frac{1}{3} : 1 : \frac{4}{3} = 1 : 3 : 4$$

13. In $\triangle ABC$, $\overline{PQ} \parallel \overline{BC}$ and $AP = 3x - 19$, $PB = x - 5$, $AQ = x - 3$, $QC = 3$ cm. Find x.

Sol. In $\triangle ABC$, $PQ \parallel BC$



$$\Rightarrow \frac{AP}{PB} = \frac{AQ}{QC}$$

(\because Basic proportionality theorem)

$$\frac{3x - 19}{x - 5} = \frac{x - 3}{3}$$

$$\Rightarrow 9x - 57 = x^2 - 8x + 15$$

$$\Rightarrow x^2 - 17x + 72 = 0$$

$$\Rightarrow (x - 8)(x - 9) = 0$$

$$\therefore x = 8 \text{ or } x = 9$$

SECTION - III

14. How many silver coins of diameter 5 cm and thickness 4 mm have to be melted to prepare a cuboid of 12 cm \times 11 cm \times 5 cm dimension?

Sol. Diameter of silver coin = 5 cm

$$\text{Radius of silver coin} = \frac{5}{2} = 2.5 \text{ cm} = r$$

$$\text{Thickness of silver coin} = 4 \text{ mm} \\ = 0.4 \text{ cm} = h$$

$$\text{Volume of each silver coin} = \pi r^2 h$$

$$= \frac{22}{7} \times (2.5)^2 \times 0.4$$

$$= \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2} \times \frac{4}{10} = \frac{11 \times 5}{7}$$

Cuboid dimensions

$$= 12 \text{ cm} \times 11 \text{ cm} \times 5 \text{ cm}$$

Volume of cuboid = lwh

$$= 12 \times 11 \times 5$$

\therefore Number of silver coins

$$= \frac{\text{Volume of cuboid}}{\text{Volume of each silver coin}}$$

$$= \frac{12 \times 11 \times 5}{\left(\frac{11 \times 5}{7}\right)} = 12 \times 7 = 84$$

OR

Incomes of the families in a locality are given. Find the mode of the data.

Income (in ₹)	1-200	201-400	401-600	601-800	801-1000
Number of families	7	10	16	12	3

Sol.

Income	Number of families	Adjusted class intervals
1-200	7	0.5-200.5
201-400	10 f_0	200.5-400.5
401-600	16 f_1	$l(400.5)-600.5$
601-800	12 f_2	600.5-800.5
801-1000	3	800.5-1000.5

Here $l = 400.5$, $f_0 = 10$, $f_1 = 16$, $f_2 = 12$, $h = 200$

$$\text{Mode} = l + \frac{(f_1 - f_0)}{2f_1 - (f_0 + f_2)} \times h$$

$$= 400.5 + \frac{(16 - 10)}{2 \cdot 16 - (10 + 12)} \times 200$$

$$= 400.5 + \left(\frac{6}{32 - 22}\right) \times 200 = 400.5 + \left(\frac{200 \times 6}{10}\right) = 400.5 + 120 = 520.5$$

15. Prove that :

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A.$$

Sol.

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A}$$

$$= \frac{-\cos^2 A + \sin^2 A}{\sin A - \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin A - \cos A)}{\sin A - \cos A}$$

$$= \sin A + \cos A$$

Hence it is proved.

OR

$$\text{Show that : } (\sec \theta - \tan \theta)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}.$$

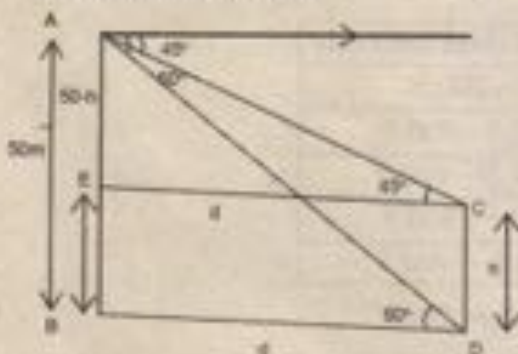
$$\text{Sol. } (\sec \theta - \tan \theta)^2 = \left(\frac{1}{\cos \theta} - \frac{\sin \theta}{\cos \theta}\right)^2$$

$$= \left(\frac{1 - \sin \theta}{\cos \theta}\right)^2$$

$$\begin{aligned}
 &= \frac{(1 - \sin \theta)^2}{\cos^2 \theta} \\
 &= \frac{(1 - \sin \theta)^2}{1 - \sin^2 \theta} \\
 &= \frac{(1 - \sin \theta)(1 - \sin \theta)}{(1 - \sin \theta)(1 + \sin \theta)} = \frac{1 - \sin \theta}{1 + \sin \theta}
 \end{aligned}$$

16. From the top of a tower of 50 m high, Neha observes the angles of depression of the top and foot of another building to be 45° and 60° respectively. Find the height of the building.

Sol. Let CD = height of the building = h
 AB = height of the tower = 50 m
 BD = EC = distance between tower and building = d m



$$\tan 45^\circ = \frac{50 - h}{d}$$

$$\Rightarrow 1 = \frac{50 - h}{d}$$

$$\Rightarrow d = 50 - h \quad \text{--- (1)}$$

$$\tan 60^\circ = \frac{50}{d}$$

$$\Rightarrow \sqrt{3} = \frac{50}{d}$$

$$\Rightarrow d = \frac{50}{\sqrt{3}}$$

$$\Rightarrow \frac{50}{\sqrt{3}} = 50 - h \quad (\because \text{From (1)})$$

$$\Rightarrow 50 = 50\sqrt{3} - h\sqrt{3}$$

$$\Rightarrow h\sqrt{3} = 50\sqrt{3} - 50$$

$$\Rightarrow h = \frac{50\sqrt{3} - 50}{\sqrt{3}} = \frac{50(\sqrt{3} - 1)}{\sqrt{3}} \text{ m}$$

OR

From the deck of 52 cards, if a card is randomly chosen, find the probability of getting a card with

- a prime number on it,
- face on it.

Sol. The number cards in a deck = 52
 Total number of outcomes = 52

- Cards with a prime number on it = {2, 3, 5, 7} (in one suit)

Number of favourable outcomes = $4 \times 4 = 16$

(\because In each suit there are 4 cards with primes)

Probability of getting a card with a prime number on it

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{16}{52} = \frac{4}{13}$$

- Number of face cards = 12

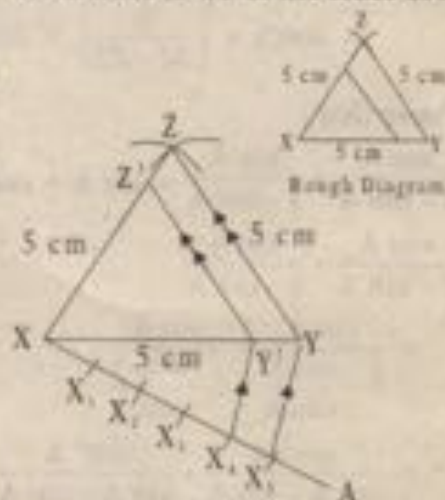
Number of favourable outcomes = 12
 Probability of getting a face

$$\text{card} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{12}{52} = \frac{3}{13}$$

17. Construct an equilateral triangle XYZ of side 5 cm and construct another triangle similar to ΔXYZ , such that each of its sides is $\frac{4}{5}$ of the sides of ΔXYZ .

Sol.



Steps of construction :

- ◆ Draw an equilateral triangle XYZ with side 5 cm,

- ◆ Draw a ray \overline{XA} such that $\angle YXA$ is an acute angle.
- ◆ Draw X_1, X_2, X_3, X_4, X_5 arcs on \overline{XA} such that $XX_1 = X_1X_2 = \dots = X_4X_5$.
- ◆ Join X_5 and Y .

- ◆ Draw a parallel line to X_5Y through X_4 to meet XY at Y' .
- ◆ Draw a parallel line to YZ through Y' to meet XZ at Z' .
- ◆ $\triangle XYZ'$ is required similar triangle.

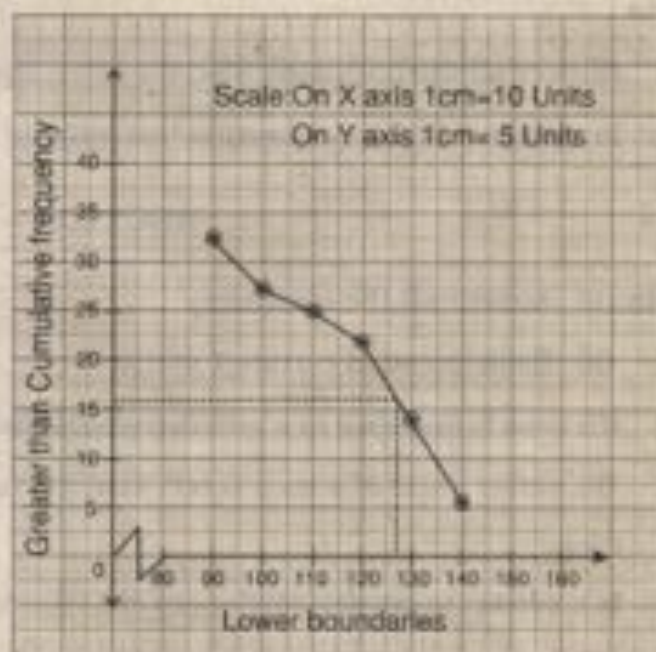
OR

Heights of the pupils of a particular school are given. Draw greater than cumulative curve and find the median height from it.

Height (in cm)	90-100	100-110	110-120	120-130	130-140	140-150
Number of pupils	5	2	3	8	8	6

Sol.

Height (in cm)	Number of pupils	Lower Boundaries	Greater than cumulative frequency (g.c.f)	(x, y)
90-100	5	90	32	(90, 32)
100-110	2	100	27	(100, 27)
110-120	3	110	25	(110, 25)
120-130	8	120	22	(120, 22)
130-140	8	130	14	(130, 14)
140-150	6	140	6	(140, 6)



$$n = 32 \Rightarrow \frac{n}{2} = \frac{32}{2} = 16$$

From graph

Median = 127.5 (approx)

PART - B

- 1) A 2) C 3) D 4) D 5) C 6) A 7) A 8) C 9) D 10) D

