CLASS - X

TELANGANA

MODEL PAPER 6

MATHEMATICS : PAPER - II

MARCH 2017

Time : 2 hours 45 min.] **Parts - A and B** [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. *In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.*
2. *Answer the Questions under '**Part – A**' on a separate answer book.*
3. *Write the answers to the Questions under '**Part – B**' on the Question paper itself and attach it to the answer book of '**Part – A**'.*

Time : 2 Hours] **PART - A** [Marks : 35

Note :

1. *Answer **all** the questions from the given **three** sections – I, II and III of **Part – A**.*
2. *In section – III, every question has internal choice. Answer **any one** alternative.*

SECTION – I

(Marks : 7 × 1 = 7)

Note : *(i) Answer **all** the questions.*

*(ii) Each question carries **1** mark.*

1. If $\sin A = \frac{1}{\sqrt{2}}$ and $\cot B = 1$, prove that $\sin(A + B) = 1$, where A and B both are acute angles.
2. The length of the minute hand of a clock is 3.5 cm. Find the area swept by minute hand in 30 minutes. $\left(\text{use } \pi = \frac{22}{7}\right)$
3. Express $\cos\theta$ in terms of $\tan\theta$.
4. From the first 50 natural numbers, find the probability of randomly selected number is a multiple of 3.
5. Write the formula to find curved surface area of a cone and explain each term in it.

6. "The median of observations, $-2, 5, 3, -1, 4, 6$ is 3.5". Is it correct ? Justify your answer.

7. If $\cos\theta = \frac{1}{\sqrt{2}}$, then find the value of $4 + \cot\theta$.

SECTION – II **(Marks : 6 × 2 = 12)**

Note : *(i) Answer **all** the questions.*

*(ii) Each question carries **2** marks.*

8. The diameter of a solid sphere is 6 cm. It is melted and recast into a solid cylinder of height 4 cm. Find the radius of cylinder.

9. Write the formula of mode for grouped data and explain each term in it.

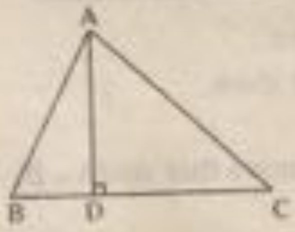
10. A person 25 mts away from a cell - tower observes the top of cell - tower at an angle of elevation $30°$. Draw the suitable diagram for this situation.

11. Find the area of the shaded region in the given figure.

ABCD is a square of side 10.5 cm.

12. One card is selected from a well - shuffled deck of 52 cards. Find the probability of getting a red card with prime number.

13. In a ΔABC, $AD \perp BC$ and $AD^2 = BD \times CD$, prove that ΔABC is a right – angled triangle.



SECTION – III **(Marks : 4 × 4 = 16)**

Note : *(i) Answer **all** the following questions.*

(ii) In this section, every question has internal choice to answer.

*(iii) Each question carries **4** marks.*

14. The length of cuboid is 12 cm, breadth and height are equal in measurements and its volume is 432 cm^3. The cuboid is cut into 2 cubes. Find the lateral surface area of each cube.

OR

Two poles are standing opposite to each other on the either side of the road which is 90 feet wide. The angle of elevation from bottom of first pole to top of second pole is 45°, the angle of elevation from bottom of second pole to top of first pole is 30°. Find the heights of poles. (use $\sqrt{3} = 1.732$).

15. A bag contains some square cards. A prime number between 1 and 100 has been written on each card. Find the probability of getting a card that the sum of the digits of prime number written on it, is 8.

OR

The daily wages of 80 workers of a factory.

Daily Wages in Rupees	500 – 600	600 – 700	700 – 800	800 – 900	900 – 1000
Number of Workers	12	17	28	14	9

Find the mean daily wages of the workers of the factory by using an appropriate method.

16. Draw a circle of diameter 6 cm from a point 5 cm away from its centre. Construct the pair of tangents to the circle and measure their length.

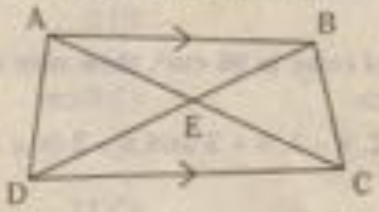
OR

The following data gives the information on the observed life span (in hours) of 90 electrical components.

Life span (in hours)	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	8	12	15	23	18	14

Draw both Ogives for the above data.

17. ABCD is a trapezium with AB ∥ DC, the diagonals AC and BD are intersecting at E. If ΔAED is similar to ΔBEC, then prove that AD = BC.



OR

Time : 30 Minutes]	PART - B	[Marks : 5

Instructions :

(i) *Answer **all** the questions.*

(ii) *Each question carries ½ mark.*

(iii) *Answers are to be written in the question paper only.*

(iv) *Marks will **not** be awarded in any case of over-writing and rewriting or erased answers.*

I. Write the CAPITAL LETTERS (A,B,C,D) showing the correct answer for the following questions in the brackets provided against them. ***(Marks : 10 × ½ = 5)***

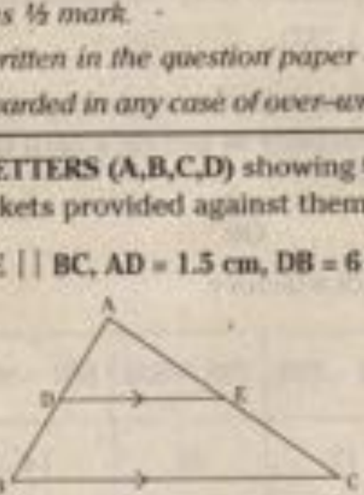
1. In the figure ΔABC, $DE \parallel BC$, AD = 1.5 cm, DB = 6 cm, AE = x cm, EC = 8 cm, then x = ... [ ]

A) 2.5 cm

B) 2 cm

C) 3 cm

D) 3.5 cm



2. The length of a tangent to a circle from a point P is 12 cm and the radius of the circle is 5 cm, then the distance from point P to the centre of the circle is .... [ ]

A) 11 cm B) 10 cm C) 13 cm D) 14 cm

3. $\tan 36^\circ . \tan 54^\circ + \sin 30^\circ$ = ... [ ]

A) $\frac{3}{2}$ B) $\frac{1}{2}$ C) 2 D) $\frac{2}{3}$

4. If $\Delta ABC \sim \Delta DEF$ and area (ΔABC) : area (ΔDEF) = 49 : 100.
Then DE : AB = ... [ ]

A) 9 : 10 B) 10 : 7 C) 10 : 9 D) 7 : 10

5. On random selection, the probability of getting a composite number among the numbers from 51 to 100. [ ]

A) $\frac{4}{5}$ B) $\frac{1}{5}$ C) $\frac{3}{5}$ D) $\frac{2}{5}$

6. If $\sin A = \frac{24}{25}$, then sec A = ... [ ]

A) $\frac{7}{25}$ B) $\frac{25}{7}$ C) $\frac{24}{7}$ D) $\frac{7}{24}$

7. 3, 2, 4, 3, 5, 2, x, 6. If the mode of this data is 3, then x = ... [ ]

A) 4 B) 3 C) 2 D) 5

8. If the total surface area of cube is 96 cm^3, then side of cube is .... [ ]

A) 3 cm B) 5 cm C) 6 cm D) 4 cm

9. For the terms, x + 1, x + 2, x – 1, x + 3 and x – 2 ($x \in N$), if the median of the data is 12, then x = ... [ ]

A) 9 B) 10 C) 11 D) 13

10. Let E and $\bar{E}$ be the complementary events.
If P ($\bar{E}$) = 0.65, then P (E) = ... [ ]

A) 0.40 B) 0.45 C) 0.35 D) 0.30

SOLUTIONS

PART - A

SECTION - I

1. If $\sin A = \frac{1}{\sqrt{2}}$ and $\cot B = 1$, prove that $\sin(A + B) = 1$, where A and B both are acute angles.

Sol. $\sin A = \frac{1}{\sqrt{2}} = \sin 45^\circ \Rightarrow A = 45^\circ$

$\cot B = 1 = \cot 45^\circ \Rightarrow B = 45^\circ$

L.H.S $\sin(A + B) = \sin(45^\circ + 45^\circ)$

$= \sin 90^\circ$

$= 1 =$ R.H.S

$\therefore$ L.H.S = R.H.S

2. The length of the minute hand of a clock is 3.5 cm. Find the area swept by minute hand in 30 minutes. $\left(\text{use } \pi = \frac{22}{7}\right)$

Sol. Angle subtended by the minutes hand in 30 min $(x^\circ) = 180^\circ$

Length of the minute's hand (r)

$= 3.5$ cm

Area swept by minutes hand in 30 min

= Area of the sector

$= \frac{x^\circ}{360^\circ} \times \pi r^2$

$= \frac{180}{360} \times \frac{22}{7} \times (3.5)^2$

$= 19.25 \text{ cm}^2$

3. Express $\cos\theta$ in terms of $\tan\theta$.

Sol. $\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{1 + \tan^2\theta}}$

4. From the first 50 natural numbers, find the probability of randomly selected number is a multiple of 3.

Sol. Multiples of 3 from 1 to 50

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48

No. of favourable outcomes to get multiples of 3 from 1 to 50 = 16

Probability of getting multiple of 3 from 1 to 50

$= \frac{\text{No.of favourable outcomes}}{\text{No.of total outcomes}} = \frac{16}{50} = \frac{8}{25}$

5. Write the formula to find curved surface area of a cone and explain each term in it.

Sol. Curved surface area of the cone = $\pi r l$

r = radius of the cone

l = slant height of the cone

6. "The median of observations, – 2, 5, 3, –1, 4, 6 is 3.5". Is it correct ? Justify your answer.

Sol. Yes.

Given data : –2, 5, 3, –1, 4, 6

Ascending order : –2, –1, 3, 4, 5, 6

Median $= \frac{3+4}{2} = \frac{7}{2} = 3.5$

7. If $\cos\theta = \frac{1}{\sqrt{2}}$, then find the value of $4 + \cot\theta$.

Sol. $\cos\theta = \frac{1}{\sqrt{2}} = \cos 45^\circ \Rightarrow \theta = 45^\circ$

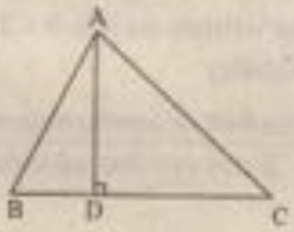
$4 + \cot\theta = 4 + \cot 45^\circ = 4 + 1 = 5$

Probability of getting red card with prime number

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of total outcomes}}$$

$$= \frac{8}{52} = \frac{2}{13}$$

13. In a ΔABC, $AD \perp BC$ and $AD^2 = BD \times CD$, prove that ΔABC is a right – angled triangle.



Sol. Given : In ΔABC, $AD \perp BC$

and $AD^2 = BD \times CD$

RTP : In ΔABC is a

right – angled triangle

Proof : $AD^2 = BD \times CD \Rightarrow \frac{AD}{BD} = \frac{CD}{AD}$

$\angle ADB = \angle CDA = 90^\circ$

$\therefore \Delta ADB \sim \Delta CDA$

($\because$ S.A.S Similarity)

In ΔABD, $\angle BAD + \angle ABD = 90^\circ$

($\because \angle ADB = 90^\circ$)

In ΔACD, $\angle DAC + \angle ACD = 90^\circ$

($\because \angle ADC = 90^\circ$)

$\angle BAD = \angle ACD$

($\because$ pair of corresponding angles)

$\angle ABD = \angle DAC$

($\because$ pair of corresponding angles)

$\therefore \angle BAD + \angle DAC = 90^\circ$

$\angle BAC = 90^\circ$

ΔABC is a right – angled triangle.

SECTION – III

14. The length of cuboid is 12 cm, breadth and height are equal in measurements and its volume is 432 cm³. The cuboid is cut into 2 cubes. Find the lateral surface area of each cube.

Sol. Length of cuboid (l) = 12 cm

Let breadth of cuboid (b) = x cm

Height of cuboid (h) = x cm

$\therefore$ Volume of the cuboid = lbh

$= 12 \times x \times x = 12x^2\ cm^3$

Volume of cuboid = 432 cm³ ($\because$ given)

$\Rightarrow 12x^2 = 432$

$$\Rightarrow x^2 = \frac{432}{12} = 36 = 6^2$$

$\therefore$ Breadth = 6cm, Height = 6cm

Cuboid cut into two cubes $\Rightarrow$ length of the each cube should be 6 cm

$\Rightarrow$ Lateral surface area of cube

$= 4(\text{side})^2$

$= 4(6)^2 = 144\ cm^2$

OR

Two poles are standing opposite to each other on the either side of the road which is 90 feet wide. The angle of elevation from bottom of first pole to top of second pole is 45°, the angle of elevation from bottom of second pole to top of first pole is 30°. Find the heights of poles. (use $\sqrt{3} = 1.732$).

Sol. Let AB = x and CD = y

In ΔABD, $\tan 30^\circ = \frac{x}{90}$

$x = 90 \times \tan 30^\circ$

$$\Rightarrow x = 90 \times \frac{1}{\sqrt{3}} = \frac{90\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 30\sqrt{3}$$

$= 30(1.732)$

$= 51.960$

$= 51.96$ ft.

In Δ BDC, $\tan 45° = \frac{y}{90}$

$\Rightarrow y = 90 \times \tan 45°$

$\Rightarrow y = 90 \times 1$

$\Rightarrow y = 90$ ft

$\therefore$ Height of the first pole = 51.96 ft

Height of the second pole = 90ft

15. A bag contains some square cards. A prime number between 1 and 100 has been written on each card. Find the probability of getting a card that the sum of the digits of prime number written on it, is 8.

Sol. Number of prime number between 1 to 100 = 25

Card that the sum of the digits of prime number written in it is 8 = 17, 71, 53

Number of cards that sum of the digits of prime written on it is 8 = 3

$\therefore$ Probability

$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$

$= \frac{3}{25}$

OR

The daily wages of 80 workers of a factory.

Daily Wages in Rupees	500 - 600	600 - 700	700 - 800	800 - 900	900 - 1000
Number of Workers	12	17	28	14	9

Find the mean daily wages of the workers of the factory by using an appropriate method.

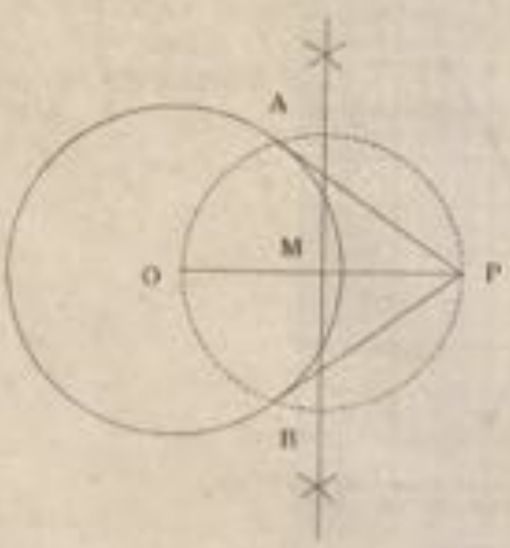
Sol.

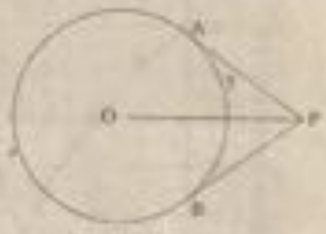
Daily Wages (in Rupees)	Number of workers(f_i)	Mid values (x_i)	f_ix_i
500 - 600	12	550	6600
600 - 700	17	650	11050
700 - 800	28	750	21000
800 - 900	14	850	11900
900 - 1000	9	950	8550
	80		59100

Mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{59100}{80} =$ Rs. 738.75

16. **Draw a circle of diameter 6 cm from a point 5 cm away from its centre. Construct the pair of tangents to the circle and measure their length.**

Sol.





Rough Diagram

Construction steps :

1) Draw a circle of 3 cm radius.
2) Mark a point P externally which is 5cm away from the centre "O".
3) Join the centre "O" and an external point "P".
4) Draw the perpendicular bisector of OP.
5) Name the midpoint as M and take M as centre and OM as radius draw a circle.
6) Name the points of intersection of the circle with constructed circle as A and B and join the points P, A and P, B.
7) PA and PB are required tangents.
8) PA = PB = 4 cm.

OR

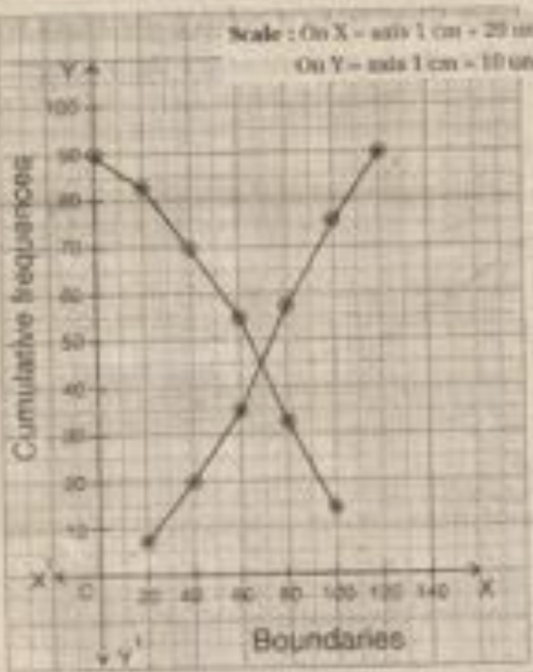
The following data gives the information on the observed life span (in hours) of 90 electrical components.

Life span (in hours)	**0 - 20**	**20 - 40**	**40 - 60**	**60 - 80**	**80 - 100**	**100 - 120**
Frequency	**8**	**12**	**15**	**23**	**18**	**14**

Draw both Ogives for the above data.

Sol.

Life span (in hrs.)	Freq - uency	Upper boundary	l.c.f.	(x, y)	Lower boundary	m.c.f.	(x, y)
0 - 20	8	20	8	(20, 8)	0	90	(0, 90)
20 - 40	12	40	20	(40, 20)	20	82	(20, 82)
40 - 60	15	60	35	(60, 35)	40	70	(40, 70)
60 - 80	23	80	58	(80, 58)	60	55	(60, 55)
80 - 100	18	100	76	(100, 76)	80	32	(80, 32)
100 - 120	14	120	90	(120, 90)	100	14	(100, 14)



17. **ABCD is a trapezium with AB ∥ DC, the diagonals AC and BD are intersecting at E. If △AED is similar to △BEC, then prove that AD = BC.**

Sol. In Trapezium ABCD, AB ∥ CD.

△ ABD, △ ABC lies on the same base AB

and between same parallels AB and CD area of △ ABD = area of △ ABC

area of △ABE + area of △ AED = area of △ ABE + area of △ BEC

∴ area of △ AED = area of △ BEC

and △ AED ~ △ BEC (∵ given)

$$\frac{\text{area of } \triangle AED}{\text{area of } \triangle BEC} = \frac{AD^2}{BC^2}$$

(∵ The ratio of area of similar triangles = the ratio of squares of corresponding sides)

$$1 = \frac{AD^2}{BC^2}$$

∴ AD = BC

OR

Prove that

$$(1+\tan^2\theta)+\left(1+\frac{1}{\tan^2\theta}\right)=\frac{1}{\sin^2\theta-\sin^4\theta}.$$

Sol. L.H.S : $(1+\tan^2\theta)+\left(1+\frac{1}{\tan^2\theta}\right)=(1+\tan^2\theta)+\left(\frac{\tan^2\theta+1}{\tan^2\theta}\right)$

$$=(1+\tan^2\theta).\left(1+\frac{1}{\tan^2\theta}\right)$$

$$=(1+\tan^2\theta).\left(\frac{\tan^2\theta+1}{\tan^2\theta}\right)$$

$$=\frac{(1+\tan^2\theta)^2}{\tan^2\theta}$$

$$=\frac{(\sec^2\theta)^2}{\tan^2\theta}\quad(\because \sec^2\theta=1+\tan^2\theta)$$

$$=\frac{1}{\cos^4\theta}\times\frac{\cos^2\theta}{\sin^2\theta}$$

$$=\frac{1}{\sin^2\theta.\cos^2\theta}$$

R.H.S : $\frac{1}{\sin^2\theta-\sin^4\theta}=\frac{1}{\sin^2\theta(1-\sin^2\theta)}$

$$=\frac{1}{\sin^2\theta.\cos^2\theta}\ (\because 1-\sin^2\theta=\cos^2\theta)$$

$\therefore$ L.H.S = R.H.S

PART - B

1. B 2. C 3. A 4. B 5. A 6. B 7. B 8. D 9. C 10. C

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