CLASS - X

TELANGANA

A GRADE

MODEL PAPER 8

MATHEMATICS : PAPER - II

MARCH 2016

Time : 2.45 Hours] Parts - A and B [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.
2. Answer the Questions under **Part – A** on a separate answer book.
3. Write the answer to the questions under **Part – B** on the question paper itself and attach it to the answer book of **Part – A**.

Time : 2.15 Hours] **PART - A** [Max. Marks : 35

Note :

1. Answer **all** the questions from the given **three** sections I, II and III of **Part – A**.
2. In section – III, every question has internal choice. Answer **any one** alternative.

SECTION – I

$7 \times 1 = 7$

Note : (i) Answer **all** the following questions.

(ii) Each question carries **1** Mark.

1. In the given figure, $\Delta ABC \sim \Delta ADE$, then find the value of 'x'.

2. Find the probability of getting a sum of the numbers on them is 7, when two dice are rolled at a time.
3. If $\tan\theta = \sqrt{3}$ (where θ is acute), then find the value of $1 + \cos\theta$.
4. "A conical solid block is exactly fitted inside the cubical box of side 'a', then the volume of conical solid block is $\frac{4}{3}\pi a^3$." Is this statement true ? Justify your answer.

5. If the surface area of a hemisphere is 'S', then express 'r' in terms of 'S'.
6. Write the formula to find the median for grouped data and explain each term.
7. "If the angle of elevation of Sun increases from 0° to 90°, then the length of shadow of a tower decreases." Is this statement true ? Justify your answer.

SECTION – II

$6 \times 2 = 12$

Note : (i) Answer **all** the following questions.

(ii) Each question carries **2** Marks.

8. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$, (where θ is acute).
9. ABC is an isosceles triangle and $\angle B = 90°$, then show that $AC^2 = 2AB^2$.
10. Find the volume and surface area of a sphere of radius 42 cm $\left(\pi = \frac{22}{7}\right)$.
11. If $\tan(A + B) = 1$, and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B < 90°$ and $A > B$; find A and B.
12. A solid metallic ball of volume 64 cm^3 melted and made into a solid cube. Find the side of the solid cube.
13. A boat has to cross a river. It crosses the river by making an angle of 60° with the bank of the river due to the stream of the river and travels a distance of 450m to reach the another side of the river. Draw the diagram for this data.

SECTION – III

$4 \times 4 = 16$

Note : (i) Answer **all** the following questions.

(ii) In this section, every question has internal choice.

(iii) Answer **anyone** alternative.

(iv) Each question carries **4** Marks.

14. a) A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of red ball, find the number of blue balls in the bag.

OR

b) Evaluate : $\frac{\tan^2 60° + 4\cos^2 45° + 3\sec^2 30° + 5\cos^2 90°}{\text{cosec}\, 30° + \sec 60° - \cot^2 30°}$

15. a) Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in Rupees	200 – 250	250 – 300	300 – 350	350 – 400	400 – 450
Number of Workers	6	8	14	10	12

Find the mean daily wages of the workers in the factory by using step-deviation method.

OR

b) Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct a pair of tangents to the circle. Find the lengths of tangents.

16. a) The following table gives production yield per hectare of Wheat of 100 farms of a village.

Production yield (Quintal/Hec.)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farmers	2	24	16	8	38	12

Draw both Ogives for the above data. Hence obtain the median production yield.

OR

b) Construct a triangle of sides 5cm, 6cm and 7cm, then construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

17. a) DWACRA is supplied cuboidal shaped wax block with measurements 88cm × 42cm × 35 cm. From this how many number of cylindrical candles of 2.8 cm diameter and 8 cm of height can be prepared ?

OR

b) Two poles of equal heights are standing opposite to each other, on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of top of the poles are 60° and 30° respectively. Find the height of the poles.

Time : 30 Mts.] **PART - B** **[Max. Marks : 5**

Instructions :

(i) Answer **all** the questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in question paper only.

(iv) Marks will **not** be awarded in any case of over - writing, rewriting or erased answers.

I. Write the **CAPITAL LETTERS (A,B,C,D)** showing the correct answer for the following questions in the brackets provided against them. **10 × ½ = 5**

1. If $\sin\theta = \cos\theta$ (where $0° < \theta < 90°$), then $\tan\theta + \cot\theta$ = ...

(A) $2\sqrt{3}$ (B) $\frac{2}{\sqrt{3}}$ (C) 2 (D) 1

2. If $\sec\theta + \tan\theta = 3$, then $\sec\theta - \tan\theta$ = ...

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$

3. **Observe the following :**

(i) The maximum number of tangents drawn from an external point to a circle is 2.

(ii) The maximum number of secants drawn from an external point to a circle is 2.

Which of the following is not true ?

(A) (i) only (B) (ii) only (C) Both (i) and (ii) (D) Neither (i) nor (ii)

4. **Let E, $\bar{E}$ be the complementary events, in a Random experiment, then which of the following is true ?**

(A) $P(E) + P(\bar{E}) = 2$ (B) $P(E) + P(\bar{E}) = 3$

(C) $P(E) + P(\bar{E}) = 1$ (D) $P(E) + P(\bar{E}) = 4$

5. **Top of a building was observed at an angle of elevation 'a' from a point, which is at a distance 'd' meters from the foot of the building. Which trigonometrical ratio should be considered for finding height of the building ?**

(A) $\tan\alpha$ (B) $\sin\alpha$ (C) $\cos\alpha$ (D) $\sec\alpha$

6. **Let $x_1, x_2, x_3, x_4, \ldots\ldots, x_n$ be the n observations and**

$\bar{x}$ be the mean of n observations, then $\sum_{i=1}^{n}(x_i - \bar{x}) = \ldots\ldots$

(A) 0 (B) $n\bar{x}$ (C) $\frac{\bar{x}}{n}$ (D) $\frac{2\bar{x}}{n}$

7. **Let r, h had *l* be the radius, height and slant height of a cone respectively, then express *l* in terms of r and h is .....**

(A) $\sqrt{h^2 - r^2}$ (B) $\sqrt{r^2 + h^2}$ (C) $\sqrt{r^2 - h^2}$ (D) $\sqrt{4r^2 + h^2}$

8. **ABC is a right angle triangle and $\angle C = 90°$, let BC = a,**

CA = b, AB = c and p be the length of the perpendicular from C on AB, then....

(A) $\frac{1}{p^2} = \frac{1}{a^2} - \frac{1}{b^2}$ (B) $\frac{1}{p^2} = \frac{1}{b^2} - \frac{1}{a^2}$ (C) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (D) $\frac{2}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. **In a ΔABC, AB = c, BC = a, AC = b and $\angle BAC = \theta$, then area of ΔABC is ....**

(where θ is acute).

(A) $\frac{1}{2}ab\sin\theta$ (B) $\frac{1}{2}ca\sin\theta$ (C) $\frac{1}{2}bc\sin\theta$ (D) $\frac{1}{2}b^2\sin\theta$

10. **Mode of the grouped data can be calculated by using the formula,**

Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$, f_1 represents .....

(A) frequency of the modal class.

(B) frequency of the class preceding the modal class.

(C) frequency of the class succeeding the modal class.

(D) cumulative frequency of the class preceding the modal class.

PART - A

SECTION – I

1. **In the given figure, $\Delta ABC \sim \Delta ADE$, then find the value of 'x'.**

Sol. $\Delta ABC \sim \Delta ADE$

$$\Rightarrow \frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$$

$BC = x, DE = 5, AE = 3, AC = 9$

By substituting $\frac{BC}{DE} = \frac{AC}{AE}$

$$\therefore \frac{x}{5} = \frac{9}{3} \Rightarrow x = \frac{9 \times 5}{3} = 15$$

$$\therefore \boxed{x = 15 \text{ cm}}$$

2. **Find the probability of getting a sum of the numbers on them is 7, when two dice are rolled at a time.**

Sol. When two dice are rolled at a time the total outcomes are $= 6^2 = 36$

Number of outcomes such that their sum of numbers on face is $7 = 6$

$\therefore$ Probability of getting sum of numbers on faces to be $7 = \frac{6}{36} = \boxed{\frac{1}{6}}$

3. **If $\tan\theta = \sqrt{3}$ (where θ is acute), then find the value of $1 + \cos\theta$.**

Sol. $\tan\theta = \sqrt{3} = \tan 60$ ($\because \theta$ is acute)

$$\Rightarrow \theta = 60$$

$$\Rightarrow 1 + \cos\theta = 1 + \cos 60 = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\therefore \boxed{1 + \cos\theta = \frac{3}{2}}$$

4. **"A conical solid block is exactly fitted inside the cubical box of side 'a', then the volume of conical solid block is $\frac{4}{3}\pi a^3$." Is this statement true ? Justify your answer.**

Sol. The cone is exactly fitted inside the cubical box.

So height of cone $= a =$ side of cube

Radius of cone $= \frac{a}{2}$

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^2 \cdot a = \frac{1}{3} \cdot \pi \cdot \frac{a^2}{4} \cdot a$$

$$= \frac{1}{3} \cdot \frac{\pi a^3}{4} = \frac{1}{12}\pi a^3$$

But to say $\frac{4}{3}\pi a^3$ is not correct.

5. If the surface area of a hemisphere is 'S', then express 'r' in terms of 'S'.

Sol. The surface area of a hemisphere

$$= 2\pi r^2 = S$$

$$\Rightarrow r^2 = \frac{S}{2\pi}$$

$$\therefore r = \sqrt{\frac{S}{2\pi}}$$

6. Write the formula to find the median for grouped data and explain each term.

Sol. Median for a grouped data :

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

where,

l - lower boundary of median class.

n - number of observations.

cf - cumulative frequency of class preceding the median class.

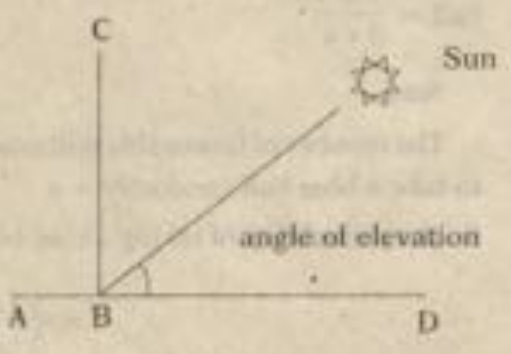
f - frequency of median class.

h - size of the median class.

7. "If the angle of elevation of Sun increases from 0° to 90°, then the length of shadow of a tower decreases." Is this statement true ? Justify your answer.

Sol. Yes, this statement is true.

We observe this in day to day life.



AD - ground

BC - lower,

AB - shadow

SECTION – II

8. Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$, (where θ is acute).

Sol.

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$= \frac{\sqrt{(1-\sin\theta)^2}}{\sqrt{1-\sin^2\theta}} = \frac{1-\sin\theta}{\sqrt{\cos^2\theta}} = \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

Hence proved

9. ABC is an isosceles triangle and $\angle B = 90°$, then show that $AC^2 = 2AB^2$.

Sol. In ΔABC ; $AB = BC$

$\therefore \angle B = 90°$

$AC^2 = AB^2 + BC^2$ $(\because \angle B = 90°)$

$\therefore AC^2 = 2AB^2$

10. Find the volume and surface area of a sphere of radius 42 cm $\left(\pi = \frac{22}{7}\right)$.

Sol. Radius of the sphere (r) = 42 cm

Curved surface area (A) = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 42 \times 42 = \boxed{22,176\ cm^2}$$

Volume of sphere $V = \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 42 \times 42 \times 42$$

$$= \boxed{3,10,464\ cm^3}$$

11. If $\tan(A+B) = 1$, and $\cos(A-B) = \frac{\sqrt{3}}{2}$, $0° < A + B < 90°$ and $A > B$: find A and B.

Sol. $\tan(A+B) = 1 = \tan 45°$

$\therefore A + B = 45° \text{ —— (1)}$

$\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30°$

$\Rightarrow A - B = 30° \text{ —— (2)}$

Solving the equations (1) and (2) we get

$A + B = 45$

$A - B = 30$

$2A = 75° \Rightarrow A = \frac{75}{2} = 37.5$

then $A + B = 45$

$37.5 + B = 45$

$\Rightarrow B = 45 - 37.5 = 7.5$

So $\boxed{A = 37.5°,\ B = 7.5°}$

12. A solid metallic ball of volume 64 cm^3 melted and made into a solid cube. Find the side of the solid cube.

Sol. Volume of solid metal = volume of cube

$= 64\ cm^3$

$\therefore$ Volume of cube $= S^3 = 64\ cm^3$

$\Rightarrow S = \sqrt[3]{64} = 4$ cm

So side of the cube = 4 cm

13. A boat has to cross a river. It crosses the river by making an angle of 60° with the bank of the river due to the stream of the river and travels a distance of 450 m to reach the another side of the river. Draw the diagram for this data.

Sol.

AB - width of river

AD, BC are river banks

AC - The distance travelled in river

= 450 m

A - initial point, C - terminal point

SECTION – III

14.a) A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of red ball, find the number of blue balls in the bag.

Sol. Number of red balls present in a bag

= 5

Let the No.of blue balls = x (say)

Then the total No.of balls = 5 + x

From those (5 + x) balls in the bag

the number of favourable outcomes to take a red ball randomly = 5

So the probability of taking a red ball $= \frac{5}{5+x}$

Now

The number of favourable outcomes to take a blue ball randomly = x

So the probability of taking a blue ball

$= \frac{x}{5+x}$

From the given problem

Probability of blue bell

= (Probability of red ball) (2)

$$\frac{x}{5+x} = \left[\frac{5}{5+x}\right](2)$$

$$\therefore \frac{x}{5+x} = \frac{10}{5+x} \Rightarrow x = 10$$

$\therefore$ No. of blue balls in the bag = 10

OR

b) Evaluate :

$$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\text{cosec } 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

Sol. Put the following values in the given problem

$$\tan 60 = \sqrt{3}, \cos 45 = \frac{1}{\sqrt{2}}, \sec 30 = \frac{2}{\sqrt{3}}$$

$$\cos 90 = 0, \text{ cosec } 30 = 2, \sec 60 = 2$$

$$\cot 30 = \sqrt{3}$$

We get

$$\frac{\tan^2 60 + 4\cos^2 45 + 3\sec^2 30 + 5\cos^2 90}{\text{cosec } 30 + \sec 60 - \cot^2 30} =$$

$$= \frac{[\sqrt{3}]^2 + 4\left[\frac{1}{\sqrt{2}}\right]^2 + 3.\left[\frac{2}{\sqrt{3}}\right]^2 + 5(0)^2}{2+2-[\sqrt{3}]^2}$$

$$= \frac{3 + 4\left(\frac{1}{2}\right) + 3\left(\frac{4}{3}\right) + 5(0)}{4-3}$$

$$= \frac{3+2+4+0}{1} = \boxed{9}$$

15.a) Consider the following distribution of daily wages of 50 workers of a factory.

Daily wages in Rupees	200 – 250	250 – 300	300 – 350	350 – 400	400 – 450
Number of Workers	6	8	14	10	12

Find the mean daily wages of the workers in the factory by using step-deviation method.

Sol.

Daily wages in rupees C.I.	No.of workers f_i	x_i	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
200 - 250	6	225	- 2	- 12
250 - 300	8	275	- 1	- 8
300 - 350	14	325 A	0	0
350 - 400	10	375	1	10
400 - 450	12	425	2	24
	$\Sigma f_i = 50$			$\Sigma f_i u_i = 14$

Assumed mean (A) = 325

$\Sigma f_i u_i = 14$; $\Sigma f_i = 50$

Class interval (h) = 50

Formula for the mean in step deviation method $\bar{x} = A + \left[\frac{\Sigma f_i u_i}{\Sigma f_i} \times h\right]$

Now substituting the above values in the formula we get

$$\bar{x} = 325 + \left(\frac{14}{50} \times 50\right) = 325 + 14 = 339$$

So mean daily wage of workers = Rs. 339

OR

b) **Draw a circle of radius 5 cm. From a point 8 cm away from its centre, construct a pair of tangents to the circle. Find the lengths of tangents.**

Sol. **Steps of construction :**

1) Construct a circle with a radius of 5 cm.

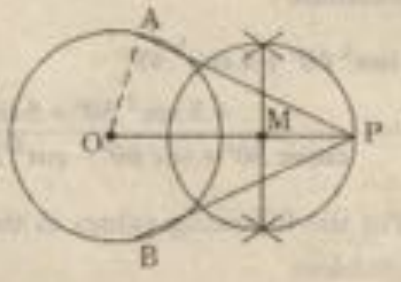
2) Trace the point 'P' in the exterior of the circle which is at a distance of '8' cm from its centre.

3) Construct a perpendicular bisector to OP which meets at M.

4) The draw a circle with a radius of MP or MO from the point M. This circle cuts the previous circle drawn from the centre 'O' at the points A and B.

5) Now join the points PA and then PB.

6) PA, PB are the required tangents which are measured 6.2 cm long.



OA = 5 cm

OP = 8 cm

AP = PB = 6.2 cm.

16.a) **The following table gives production yield per hectare of Wheat of 100 farms of a village.**

Production yield (Quintal/Hec.)	50 – 55	55 – 60	60 – 65	65 – 70	70 – 75	75 – 80
Number of farmers	2	24	16	8	38	12

Draw both Ogives for the above data. Hence obtain the median production yield.

Sol. **Part – I :** We consider upper limits of class on X-axis and cumulative frequency on Y-axis to draw more than ogive.

Production	Number of farmers f	Cumulative frequency C.F.
< 55	2	2
< 60	24	26
< 65	16	42
< 70	8	50
< 75	38	88
< 80	12	100

So the points (55, 2) (60, 26) (65, 42) (70, 50) (75, 88) and (80, 100) are to be plotted by choosing the scale on X - axis 1 cm = 50 units ; on Y – axis 1 cm = 10 units We get more than ogive

Part II

To draw less than ogive, we choose lower limits on X - axis and less than cumulative frequency on Y - axis.

Production	Number of farmers	Less than cumulative frequency
50 and above	2	100
55 and above	24	98
60 and above	16	74
65 and above	8	58
70 and above	38	50
75 and above	12	12

Now the points to be plotted on graph

= (50, 100) (55, 98) (60, 74) (65, 58) (70, 50) and (75, 12)

Scale on X - axis 1cm = 50 unit ;

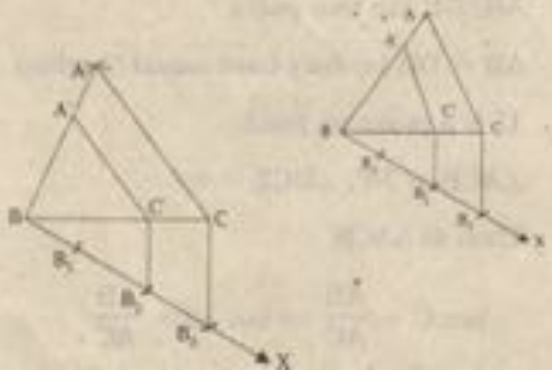
on Y – axis 1 cm = 10 units

The above two curves cross at some point. Now we draw a perpendicular line to X-axis from this point. The coordiante on X - axis (foot of perpendicular) is the median.

OR

b) Construct a triangle of sides 5cm, 6cm and 7cm, then construct a triangle similar to it, whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.

Sol. Rough Figure



Construction steps :

1) Draw a triangle ΔABC with sides AB = 5 cm, BC = 6 cm and CA = 7 cm

2) Draw a ray BX making an acute angle with BC on the side opposite to vertex A.

3) Locate 3 points B_1, B_2, B_3 on BX. So that $BB_1 = B_1B_2 = B_2B_3$.

4) Join B_3, C and draw a line through B_2 parallel to B_3 C intersecting BC at C'.

5) Draw a line through C' parallel to CA intersect AB at A'.

6) $\Delta A'B'C'$ is required triangle.

17.a) DWACRA is supplied cuboidal shaped wax block with measurements 88cm × 42cm × 35 cm. From this how many number of cylindrical candles of 2.8 cm diameter and 8 cm of height can be prepared ?

Sol. Shape of wax block = cuboid

Its length (l) = 88 cm

breath (b) = 42 cm

height (h) = 35 cm

Then the volume of wax present in block = lbh

$= 88 \times 42 \times 35\ cm^3$ — (1)

Shape of candle = cylinder

Diameter of candle = (d) = 2.8 cm

$\Rightarrow$ radius = $r = \frac{2.8}{2} = 1.4$ cm

height (h) = 8 cm

Volume of wax required to make one candle = $V = \pi r^2 h$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 8\ cm^3$

$\therefore$ Total number of candles that can be made = $\frac{\text{Total volume of block}}{\text{Volume of each candle}}$

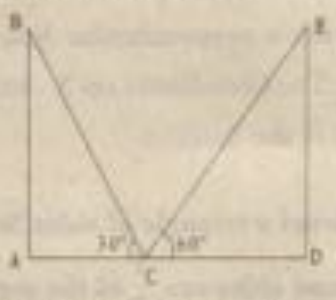
$= \frac{88 \times 42 \times 35}{\frac{22}{7} \times 1.4 \times 1.4 \times 8} = 2625$

So 2625 candles can be made with given measurements.

OR

b) Two poles of equal heights are standing opposite to each other, on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of top of the poles are 60° and 30° respectively. Find the height of the poles.

Sol. As shown in the figure



AD = width of road = 80 m.

AB, DE are two poles

AB = DE ($\because$ they have equal heights)

'C' is a point on road.

$\angle ACB = 30°$, $\angle DCE = 60°$

Then in ΔACB

$\tan C = \frac{AB}{AC} \Rightarrow \tan 30 = \frac{AB}{AC}$

$$\frac{1}{\sqrt{3}} = \frac{AB}{AC} \Rightarrow AC = AB\sqrt{3} \quad \text{— (1)}$$

In ΔCDE

$$\tan C = \frac{DE}{CD} \Rightarrow \tan 60 = \frac{DE}{CD}$$

$$\sqrt{3} = \frac{DE}{CD} \Rightarrow CD = \frac{DE}{\sqrt{3}} \quad \text{— (2)}$$

but $AC + CD = AD$

$$AB\sqrt{3} + \frac{DE}{\sqrt{3}} = 80$$

But $DE = AB$

$$\Rightarrow \quad AB\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

$$\Rightarrow \quad \frac{3AB + AB}{\sqrt{3}} = 80$$

$$\Rightarrow \quad 4AB = 80\sqrt{3}$$

$$\Rightarrow AB = \frac{80\sqrt{3}}{4} = 20\sqrt{3}$$

So height of the pole = $20\sqrt{3}$ m.

PART - B

1. C 2. A 3. B 4. C 5. A 6. A 7. B 8. C 9. C 10. A