CLASS - X

TELANGANA

A GRADE

MODEL PAPER 1

MATHEMATICS : PAPER - II

JUNE 2019

Time : 2.45 Hours] Parts - A and B [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. Read the whole question paper and understand every question thoroughly without writing anything and 15 minutes of time is allotted for this.
2. Answer the questions under **Part – A** on a separate answer book.
3. Write the answers to the questions under **Part – B** on the question paper itself and attach it to the answer book of **Part – A**
4. Answer **all** the questions from the given **Three** sections I, II and III of **Part – A.**
5. In Section III, every question has internal choice. Answer any **one** alternative.

Time : 2 hrs 15 min.] **PART - A** [Marks : 35

SECTION – I

(Marks : 7 × 1 = 7)

Note : i) Answer **all** the questions.
ii) Each question carries **1** mark.

1. Madhavi said "All squares are similar". Do you agree with her statement ? Justify your answer.
2. Ravi says "the value of tan 0° tan 1° tan 2° ..... tan 89° is zero". Do you agree with Ravi ? Give reason.
3. Write two examples for equally likely events.
4. Find the mode of the data 6, 8, 3, 6, 3, 7, 4, 6, 7, 3, 6.
5. Draw a line segment of length 7.3 cm and divide it in the ratio 3 : 4.
6. If the metallic cylinder of height 4 cm and radius 3 cm is melted and recast into a sphere, then find the radius of the sphere.
7. Write the formula for finding lateral surface area of a cylinder and explain each term in it.

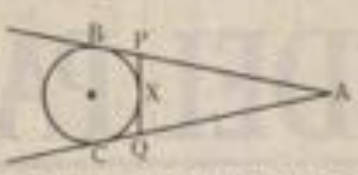
SECTION – II

(Marks : 6 × 2 = 12)

Note : i) Answer **all** the questions.
ii) Each question carries **2** marks.

8. A box contains 4 red balls, 5 green balls and P white balls. If the probability of randomly picked ball from the box to be a red ball is $\frac{1}{3}$, then find the number of white balls.
9. Prove that : $\frac{1}{\sin^2\theta} - \cot^2\theta = 1$

10. In the given figure AB, AC and PQ are tangents to a circle and AB = 6 cm. Find the perimeter of ΔAPQ.



11. If the ratio of areas of two equilateral triangles is 25 : 36, then find the ratio of heights of the triangles.
12. Find the median of first six prime numbers.
13. From the top of the building, the angle of elevation of the top of a T.V. tower is α° and the angle of depression to its (T.V. tower) foot is β°. If distance of the building from the tower is 'd' meters, draw the suitable diagram to the given data.

SECTION – III

(Marks : 4 × 4 = 16)

Note : i) Answer **all** the questions.
ii) In this section, every question has internal choice to answer.
iii) Each question carries 4 marks.

14. If the mean of the following frequency distribution is 50, then find the value of k.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	20	32	k	19

OR

In a right angle triangle, length of the hypotenuse is 6 cm more than its shortest side. The length of the other side is 3 cm less than the hypotenuse, then find the sides of right angle triangle.

15. Draw a circle of radius 6 cm and construct two tangents to the circle so that angle between the tangents is 60°.

OR

The following table gives the marks obtained by 100 students in SA-1 exams in Mathematics subject.
Draw Ogive graph of less than and greater than.

Marks	50-55	55-60	60-65	65-70	70-75	75-80
No. of students	2	8	12	24	38	16

16. A man observes top of a tower at an angle of elevation of 30°. When he walked 40 m towards the tower, the angle of elevation is changed to 60°. Find the height of the tower and distance from the first observation point to the tower.

OR

A cylindrical tank of radius 7 m has water to some level. If 110 cubes of side 7 cm are completely immersed in it, then find the rise in water level.

17. From a pack of 52 playing cards, Jacks, Queens, Kings and Aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is (i) a black queen, (ii) a red card.

OR

If $\operatorname{cosec} A = \sqrt{2}$, then find the value of $\dfrac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

Time : 30 Mts.] **PART - B** [Marks : 5

Note :

(i) Answer **all** questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in the question paper only.

(iv) Marks will **not** be awarded in any case of over-written, rewritten or erased answers.

(v) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

1. **The probability of sure event is** [ ]

   A) 0 B) $\frac{1}{2}$ C) 1 D) Undefined

2. **Sec θ =** [ ]

   A) $\sqrt{1-\cos^2\theta}$ B) $\sqrt{1-\tan^2\theta}$ C) $\tan^2\theta - 1$ D) $\sqrt{\frac{1}{1-\sin^2\theta}}$

3. **Side of a cube and diameter of a sphere are equal, then the ratio of their volume will be** [ ]

   A) 4 : π B) 6 : π C) 3 : π D) 2 : π

4. **A dice is thrown once. The probability of getting a prime number is** [ ]

   A) $\frac{1}{3}$ B) $\frac{1}{2}$ C) $\frac{2}{3}$ D) $\frac{1}{6}$

5. **A metallic sphere of radius 'r' is melted and recast into the shape of solid cylinder of radius 'r', the height of the cylinder is** [ ]

   A) 3r B) $\frac{3}{4}r$ C) $\frac{4}{3}r$ D) 4r

6. **Mean of certain number of observations is $\bar{x}$. If each observation is divided by m($m \neq 0$) and then increased by n, then the mean of new observation is** [ ]

   A) $\frac{\bar{x}}{n} + m$ B) $\bar{x} + \frac{n}{m}$ C) $\bar{x} + \frac{m}{n}$ D) $\frac{\bar{x}}{m} + n$

7. **In the given figure, PB∥CF and DP∥EF, then $\frac{AD}{DE}$ =** [ ]

   A) $\frac{1}{3}$ B) $\frac{3}{4}$

   C) $\frac{1}{4}$ D) $\frac{2}{3}$

8. **A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle of 60° with the wall. Then the height of the wall is** [ ]

   A) $15\sqrt{3}$ m B) $\frac{15\sqrt{3}}{2}$ m C) 7.5 m D) 15 m

9. **If sec θ + tan θ = x, then sec θ =** [ ]

   A) $\frac{x^2+1}{x}$ B) $\frac{x^2+1}{2x}$ C) $\frac{x^2-1}{2x}$ D) $\frac{x^2-1}{x}$

10. **At point 'P' on a circle, PQ is a tangent and 'O' is the centre of the circle. If ΔOPQ is an isosceles triangle, then ∠OQP is equal to** [ ]

    A) 90° B) 30° C) 45° D) 60°

SOLUTIONS

PART - A

SECTION – I

1. **Madhavi said "All squares are similar". Do you agree with her statement ? Justify your answer.**

A. Yes. All squares are similar.
I agree with her statement.
Because All regular polygons having the same number of sides are always similar.

2. **Ravi says "the value of tan 0°.tan 1°. tan 2°. ..... tan 89° is zero". Do you agree with Ravi ? Give reason.**

Sol. $\tan 0^\circ.\tan(90-89)^\circ.\tan(90-88)^\circ$ ..... $\tan 89^\circ$

$\tan 0^\circ.\cot 89^\circ.\cot 88^\circ$ ..... $\tan 88^\circ.\tan 89^\circ$

$\tan 0^\circ(\cot 89^\circ.\tan 89^\circ)(\cot 88.\tan 88^\circ)$ ..... $\tan 45^\circ$

$= \tan 0^\circ.1.1$ ..... $1 = 0.1 = 0$ $(\because \tan 0^\circ = 0)$

Yes. I agree with Ravi Answer.

3. **Write two examples for equally likely events.**

A. Example 1 : Tossing a coin
Head and tails have equal chances.
Example 2 : Rolling a dice.
All faces have equal chances.

4. **Find the mode of the data 6, 8, 3, 6, 3, 7, 4, 6, 7, 3, 6.**

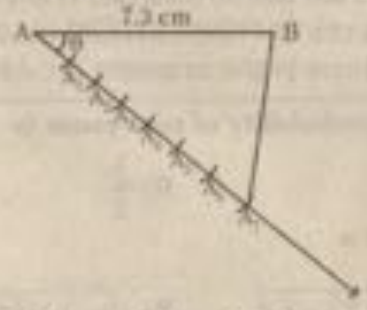
Sol. First we arrange the ascending order
3, 3, 3, 4, 6, 6, 6, 6, 7, 7, 8

x	frequency
3	3
4	1
6	4
7	2
8	1

∴ Mode (6) [having maximum frequency]

5. **Draw a line segment of length 7.3 cm and divide it in the ratio 3 : 4.**

A.



6. **If the metallic cylinder of height 4 cm and radius 3 cm is melted and recast into a sphere, then find the radius of the sphere.**

Sol. Height of cylinder = 4 cm
Radius r = 3 cm
Volume of cylinder $V = \pi r^2 h$
$= \pi \times 3^2 \times 4$
$= 36\pi$

Volume of cylinder = Volume of sphere

$$36\pi = \frac{4}{3}\pi r_s^3$$

$$r_s^3 = \frac{36\pi \times 3}{4\pi} = 27$$

$r_s = 3$ cm

∴ Radius of sphere = 3 cm.

7. **Write the formula for finding lateral surface area of a cylinder and explain each term in it.**

Sol. Lateral surface area of cylinder
L.S.A = $2\pi rh$
Here r = radius;
h = height

SECTION – II

8. A box contains 4 red balls, 5 green balls and P white balls. If the probability of randomly picked ball from the box to be a red ball is $\frac{1}{3}$, then find the number of white balls.

Sol. $P(E) = \frac{\text{No. of outcomes}}{\text{Sample Space}}$

$\frac{1}{3} = \frac{4}{4+5+P}$

$4 + 5 + P = 12 \Rightarrow P = 3$

$\therefore$ Number of white balls = 3

9. Prove that : $\frac{1}{\sin^2\theta} - \cot^2\theta = 1$

Sol. Given : $\frac{1}{\sin^2\theta} - \cot^2\theta$

$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta} \quad \left[\because \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta}\right]$

$= \frac{1-\cos^2\theta}{\sin^2\theta}$

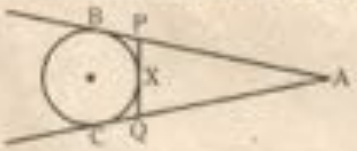
$= \frac{\sin^2\theta}{\sin^2\theta} = 1$

$[\because \sin^2\theta + \cos^2\theta = 1$

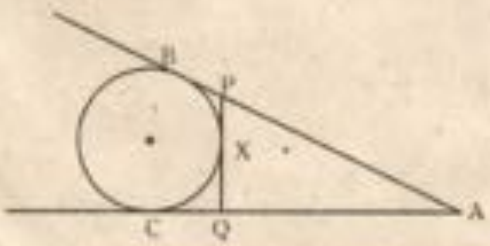
$\Rightarrow \sin^2\theta = 1 - \cos^2\theta]$

$\therefore \frac{1}{\sin^2\theta} - \cot^2\theta = 1.$

10. In the given figure AB, AC and PQ are tangents to a circle and AB = 6 cm. Find the perimeter of ΔAPQ.



Sol.



We know that tangents drawn from an external point to a circle are equal in length.

$\therefore$ AB = AC

AP + PB = AQ + QC

AP + PX = AQ + QX

($\because$ PX = BP and QX = QC)

AP + PX = AQ + QX = 6 cm

Now, Perimeter of

ΔAPQ = AP + PQ + AQ

= (AP + PX) + (QX + AQ)

= 6 + 6

= 12 cm

$\therefore$ Perimeter of ΔAPQ = 12 cm

11. If the ratio of areas of two equilateral triangles is 25 : 36, then find the ratio of heights of the triangles.

Sol. For similar triangles

Ratio of area = Ratio of sides

So, Here $\frac{25}{36} = \left(\frac{b_1}{b_2}\right)^2 = \frac{b_1^2}{b_2^2}$

$b_1 = 5, \quad b_2 = 6$

Now $\frac{\frac{1}{2}b_1h_1}{\frac{1}{2}b_2h_2} = \frac{25}{36}$

$\frac{5h_1}{6h_2} = \frac{25}{36}$

$\frac{h_1}{h_2} = \frac{5}{6}$

$\therefore h_1 : h_2 = 5 : 6$

$\therefore$ The ratio of heights of the triangles = 5 : 6

12. Find the median of first six prime numbers.

Sol. First six prime numbers

2, 3, (5), (7), 11, 13

Median of the above data,

If n is even then the median will be the average of the $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observations.

$\left(\frac{n}{2}\right)^{th}$ term $= \left(\frac{6}{2}\right) = 3^{rd}$ term

$\left(\frac{n}{2}+1\right)^{th}$ term $= \left(\frac{6}{2}+1\right)^{th}$

$= 3 + 1 = 4^{th}$ term

$$\text{Median} = \left(\frac{3^{rd}\text{ term} + 4^{th}\text{ term}}{2}\right)$$

$$= \frac{5+7}{2} = \frac{12}{2} = 6$$

∴ Median of first six prime number is 6.

13. From the top of the building, the angle of elevation of the top of a T.V. tower is $\alpha°$ and the angle of depression to its (T.V. tower) foot is $\beta°$. If distance of the building from the tower is 'd' meters, draw the suitable diagram to the given data.

A.

SECTION – III

14. If the mean of the following frequency distribution is 50, then find the value of k.

Class	0-20	20-40	40-60	60-80	80-100
Frequency	17	20	32	k	19

Sol.

Class	x	Frequency	f(x)
0-20	10	17	$10 \times 17 = 170$
20-40	30	20	$30 \times 20 = 600$
40-60	50	32	$50 \times 32 = 1600$
60-80	70	k	70k
80-100	90	19	$90 \times 19 = 1710$
		$\Sigma f_i = 88+k$	$\Sigma f_i x_i = 4080 + 70k$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$50 = \frac{4080 + 70k}{88 + k}$$

$50(88+k) = 4080 + 70k$

$4400 + 50k = 4080 + 70k$

$4400 - 4080 = 70k - 50k$

$320 = 20k$

$\frac{320}{20} = k$

$\therefore k = 16$

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$= 3 + 1 = 4^{th}$ term

$$\text{Median} = \left(\frac{3^{rd}\text{ term} + 4^{th}\text{ term}}{2}\right)$$

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13. From the top of the building, the angle of elevation of the top of a T.V. tower is $\alpha°$ and the angle of depression to its (T.V. tower) foot is $\beta°$. If distance of the building from the tower is 'd' meters, draw the suitable diagram to the given data.

A.

SECTION – III

14. If the mean of the following frequency distribution is 50, then find the value of k.

Class	0-20	20-40	40-60	60-80	80-100
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Sol.

Class	x	Frequency	f(x)
0-20	10	17	$10 \times 17 = 170$
20-40	30	20	$30 \times 20 = 600$
40-60	50	32	$50 \times 32 = 1600$
60-80	70	k	70k
80-100	90	19	$90 \times 19 = 1710$
		$\Sigma f_i = 88+k$	$\Sigma f_i x_i = 4080 + 70k$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$50 = \frac{4080 + 70k}{88 + k}$$

$50(88+k) = 4080 + 70k$

$4400 + 50k = 4080 + 70k$

$4400 - 4080 = 70k - 50k$

$320 = 20k$

$$\frac{320}{20} = k$$

$\therefore k = 16$

OR

In a right angle triangle, length of the hypotenuse is 6 cm more than its shortest side. The length of the other side is 3 cm less than the hypotenuse, then find the sides of right angle triangle.

Sol. Let the shortest side of right angled triangle be x cm.

Hypotenuse = (x + 6) cm given

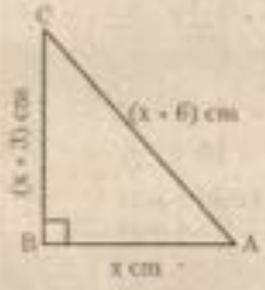
Third side = (x + 6 – 3)

= (x + 3) cm given

AC = (x + 6) cm, AB = x cm,

BC = (x + 3) cm

In a right angled triangle ABC



$AC^2 = AB^2 + BC^2$

$(x + 6)^2 = x^2 + (x + 3)^2$

$x^2 + 12x + 36 = x^2 + x^2 + 6x + 9$

$x^2 - 6x - 27 = 0$

$x^2 - 9x + 3x - 27 = 0$

$x(x - 9) + 3(x - 9) = 0$

$(x + 3)(x - 9) = 0$

$x = 9; x = -3$

(we can't consider –ve values)

Shortest side x = 9 cm

Hypotenuse = x + 6 = 9 + 6 = 15 cm

Third side = (x + 6 – 3)

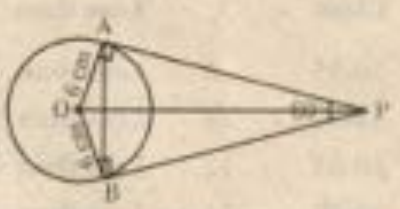
= x + 3 = 9 + 3 = 12 cm

15. Draw a circle of radius 6 cm and construct two tangents to the circle so that angle between the tangents is 60°.

Sol. To begin let us consider a circle with centre 'O' and radius 6 cm.

Let PA and PB are two tangents draw from a point 'P' outside the circle and the angle between them is 60°.

In this ∠APB = 60°. Join OP.



As we know, OP is the bisector of ∠APB, then $\angle OAP = \angle OPB = \frac{60^\circ}{2}$

$= 30^\circ$

($\because \Delta OAP \cong \Delta OBP$)

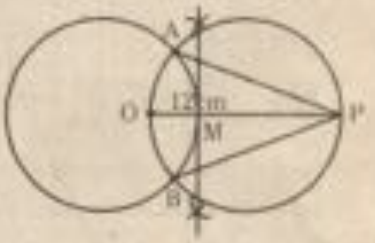
Now in ΔOAP,

$\sin 30^\circ = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{OA}{OP}$

$\frac{1}{2} = \frac{6}{OP}$ (From trigonometric ratio)

OP = 12 cm

Now we can draw a circle of radius 6 cm with centre 'O'. We then mark a point at a distance of 12 cm from the centre of the circle. Join OP and complete the construction.



Hence PA and PB are the required pair of tangents to the given circle.

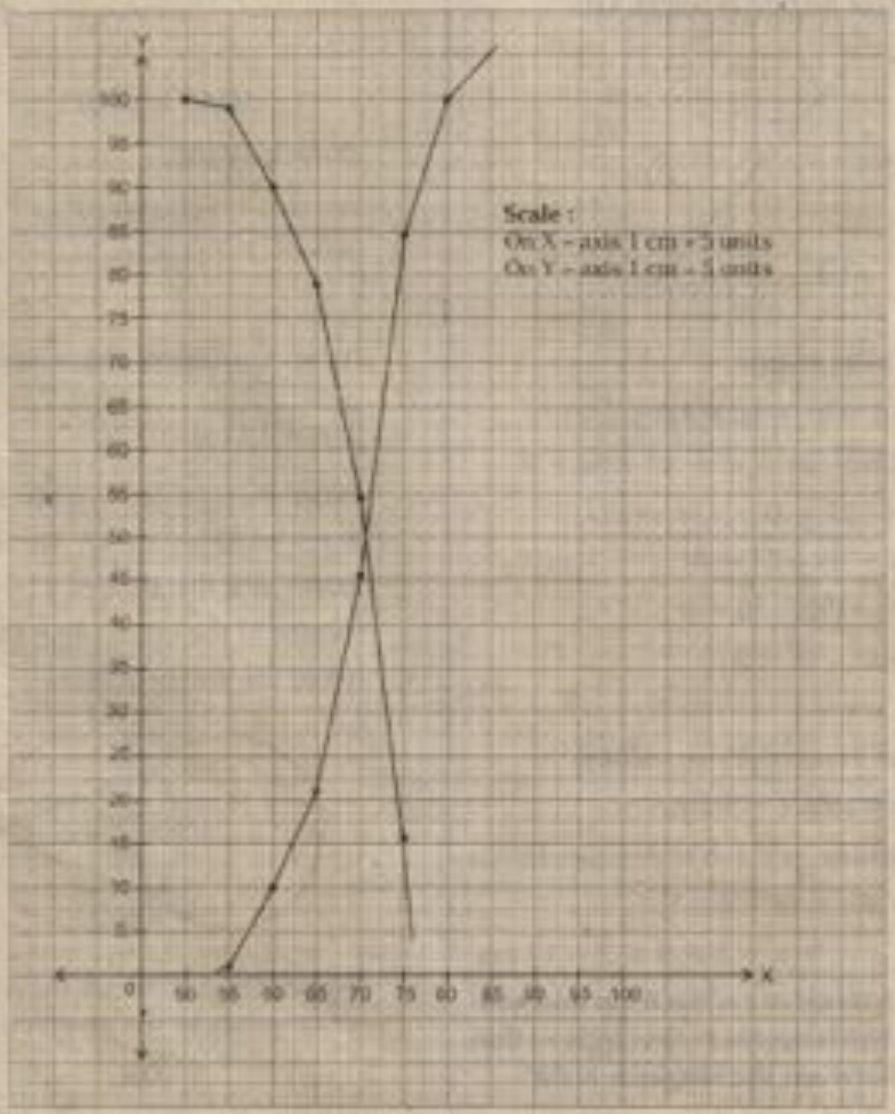
OR

The following table gives the marks obtained by 100 students in SA-1 exams in Mathematics subject.
Draw Ogive graph of less than and greater than.

Marks	50-55	55-60	60-65	65-70	70-75	75-80
No. of students	2	8	12	24	38	16

Sol. The given data is to be changed to more than frequency distribution type.

Class	f_i	Less than	c.f	More than	c.f
50-55	2	Less than 55	2	More than 50	100
55-60	8	Less than 60	10	More than 55	98
60-65	12	Less than 65	22	More than 60	90
65-70	24	Less than 70	46	More than 65	78
70-75	38	Less than 75	84	More than 70	54
75-80	16	Less than 80	100	More than 75	16

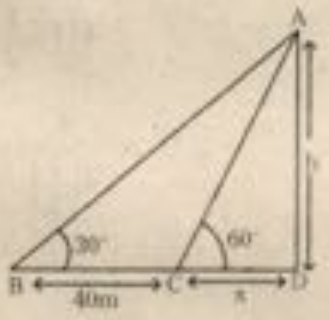


16. A man observes top of a tower at an angle of elevation of 30°. When he walked 40 m towards the tower, the angle of elevation is changed to 60°. Find the height of the tower and distance from the first observation point to the tower.

Sol. Given height of the tower we will find

BC = 40 m; CD = x m

Height of the tower = hm



From figure $\tan 30° = \frac{h}{x+40}$

$$\frac{1}{\sqrt{3}} = \frac{h}{x+40}$$

$$x + 40 = \sqrt{3}h$$

$$x = \sqrt{3}h - 40 \quad \text{—— (1)}$$

$$\tan 60° = \frac{h}{x}$$

$$\sqrt{3} = \frac{h}{\sqrt{3}h - 40}, \quad [\because \text{from (1)}]$$

$$3h - 40\sqrt{3} = h$$

$$2h = 40\sqrt{3}$$

$$h = 20\sqrt{3}\ m$$

∴ Height of the tower = $20\sqrt{3}$ m.

$$x = \sqrt{3}h - 40$$

$$= \sqrt{3} \times 20\sqrt{3} - 40$$

$$= 3 \times 20 - 40$$

$$= 60 - 40 = 20m.$$

Distance from the first observation point to the tower.

BD = x + 40 = 20 + 40 = 60m.

OR

A cylindrical tank of radius 7 m has water to some level. If 110 cubes of side 7 cm are completely immersed in it, then find the rise in water level.

Sol. Volume of the cylinder

= 110 × cube of a side.

$$\pi r^2 h = 110 r^3$$

$$\frac{22}{7} \times 7 \times 7 \times h = 110 \times (0.07)^3$$

$$h = \frac{110 \times 0.07 \times 0.07 \times 0.07}{22 \times 7}$$

$$= \frac{0.03773}{154}$$

$$h = 0.000245\ m$$

∴ Rise in water level h = 0.000245 m.

17. From a pack of 52 playing cards, Jacks, Queens, Kings and Aces of red colour are removed. From the remaining, a card is drawn at random. Find the probability that the card drawn is (i) a black queen, (ii) a red card.

Sol. Probability of black queen = $\frac{1}{26}$

Probability of red card = $\frac{1}{2}$

n(S) = 52

Probability of black queen n(a) = 2

$$p(a) = \frac{n(a)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

Probability of a red card n(b) = 26

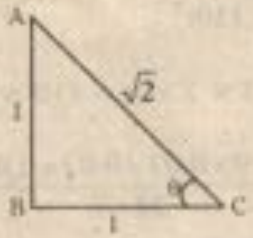
$$P(b) = \frac{n(b)}{n(S)} = \frac{26}{52} = \frac{1}{2}$$

OR

If $\operatorname{cosec} A = \sqrt{2}$, then find the value of $\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)}$.

Sol. Given $\operatorname{cosec} A = \sqrt{2}$

$$= \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{\sqrt{2}}{1}$$



$$\sin A = \frac{1}{\sqrt{2}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{1}{1}$$

$$= 1$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{1} = 1$$

$$\frac{2\sin^2 A + 3\cot^2 A}{4(\tan^2 A - \cos^2 A)} = \frac{2\left(\frac{1}{\sqrt{2}}\right)^2 + 3(1)}{4\left(1 - \left(\frac{1}{\sqrt{2}}\right)^2\right)}$$

$$= \frac{2\left(\frac{1}{2}\right) + 3}{4\left(1 - \frac{1}{2}\right)}$$

$$= \frac{4}{4\left(\frac{1}{2}\right)} = 2$$

PART - B

1) C 2) D 3) B 4) B 5) C 6) D 7) A 8) B 9) B 10) C

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