CLASS - X

TELANGANA

A GRADE

MODEL PAPER 3

MATHEMATICS : PAPER - II

JUNE 2018

Time : 2.45 Hours] | Parts - A and B | [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. Read the whole question paper and understand every question thoroughly without writing anything and 15 minutes of time is allotted for this.
2. Answer the questions under **Part - A** on a separate answer book.
3. Write the answers to the questions under **Part - B** on the question paper itself and attach it to the answer book of **Part - A**.
4. Answer **all** the questions from the given three sections I, II and III of **Part - A.**
5. In Section III, every question has internal choice. Answer any **one** alternative.

Time : 2 hrs 15 min.] **PART - A** **[Marks : 35**

SECTION - I

(Marks : 7 × 1 = 7)

Note : i) Answer **all** the questions.
ii) Each question carries **1** mark.

1. When a dice is rolled, find the probability of getting odd prime number.
2. Write the formula to find the volume of a cone and explain each term in it.
3. Find the volume of liquid that hemispherical bowl can hold, where radius of the bowl is 4.2 cm.
4. Prove that $4 \tan^2 45^\circ - \text{cosec}^2 30^\circ + \cos^2 30^\circ = \frac{3}{4}$.
5. Find the length of the tangent to circle from a point 13 cm away from the centre of a circle of radius 5 cm.
6. Find the mean of prime numbers which are less than 30.
7. Using the figure given of ΔABC, prove that $\sin^2 \theta + \cos^2 \theta = 1$.

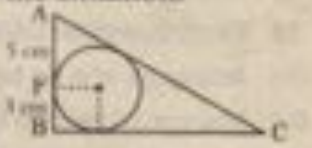
SECTION - II

(Marks : 6 × 2 = 12)

Note : i) Answer **all** the questions.
ii) Each question carries **2** marks.

8. If $\text{cosec}(A + B) = 1$ and $\cot(A - B) = \sqrt{3}$, $0^\circ < A + B \leq 90^\circ$, $A > B$, then find A and B.
9. The diameter of the base of a right circular cone is 12 cm and volume 376.8 cm^3. Find its height ($\pi = 3.14$).

10. A bag contains 7 red, 5 white and 6 black balls. A ball is drawn from the bag at random, find the probability that the ball drawn is not black.
11. In ΔABC, D and E are points on AB and AC respectively. If AB = 14 cm; AD = 3.5 cm, AE = 2.5 cm and AC = 10 cm, show that DE || BC.
12. The angle of elevation of the top of a tower from a point on the ground, which is 50 m away from the foot of the tower is 45°. Draw the diagram for the situation.
13. A circle of radius 3 cm is inscribed in a triangle ABC and AF = 5 cm, BF = 3 cm as shown in the figure.



Somu said that the measure of the side AC is 17 cm. Do you agree ? Give reasons.

SECTION – III

(Marks : 4 × 4 = 16)

Note : i) Answer **all** the questions.

ii) This section has internal choice to answer.

iii) Each question carries 4 marks.

14. A metallic sphere of diameter 30 cm is melted and recast into a cylinder of radius 10 cm. Find the height of the cylinder.

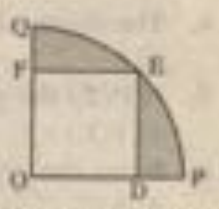
OR

Two boys on either side of their school building of 20 m height observes its top at the angles of elevation 30° and 60° respectively. Find the distance between two boys.

15. Prove that : $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

OR

A square ODEF is inscribed in a quadrant OPEQ of circle and $OD = 14\sqrt{2}$ cm. Aarthi said that "the area of shaded region is 224 cm^2." Do you agree ? Give reasons.



16. The following table shows that ages of the patients admitted in a hospital during a year.

Age in years	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Patients	8	15	25	27	18	7

Draw a less than Ogive curve for the above data.

OR

Construct a triangle ABC in which AB = 5 cm, BC = 7 cm and $\angle ABC = 50°$, then construct a triangle similar to it, whose sides are 4/5 of the corresponding sides of first triangle.

17. The below distribution gives the weight of 40 students in a class. Find the median weight of the students.

Weight in kg.	30 - 35	35 - 40	40 - 45	45 - 50	50 - 55	55 - 60
No. of students	4	5	10	8	8	5

OR

Suppose you drop a dice at random on the circular region of diameter 28 cm as shown in the figure. What is the probability that it will land inside the rectangle ?

Time : 30 Mts.] **PART - B** [Marks : 5

SECTION – IV

Note :

(i) Answer **all** the questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in the question paper only.

(iv) Marks will **not** be awarded in any case of over-written, rewritten or erased answers.

(v) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

1. From a set of single digit natural numbers, if a number chosen at random, then the probability that the number chosen is a multiple of 2, is [ ]

   A) $\frac{4}{9}$ B) $\frac{1}{3}$ C) $\frac{9}{4}$ D) $\frac{2}{5}$

2. $2 - 2\sin^2 60^\circ =$ [ ]

   A) $\sin 60^\circ$ B) $\tan 60^\circ$ C) $\cos 60^\circ$ D) $\sec 60^\circ$

3. If 14 is deleted from the data 12, 14, 15, 16, 17, 18, 19 and 20, then the median increases by [ ]

   A) 1 B) 1.5 C) 2 D) 0.5

4. The mean of the first eight multiples of 3 is [ ]

   A) 8 B) 13.5 C) 13 D) 27

5. If P(E) the probability of an event, then [ ]

   A) $P(E) \geq 1$ B) $P(E) \leq 0$ C) $0 \leq P(E) \leq 1$ D) $P(E) \leq 1$

6. From the given figure, $\angle XOY = 130^\circ$, then $\angle XPO =$ [ ]

   A) 65°
   B) 35°
   C) 25°
   D) 55°

7. The value of $\cos 15^\circ \times \cos 45^\circ \times 2\,\text{cosec}\, 75^\circ$ is [ ]

   A) $\frac{2}{\sqrt{3}}$ B) $\frac{\sqrt{3}}{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\sqrt{2}$

8. The volume of a cylinder is given by the formula $\pi r^2 h$, here 'h' represents [ ]

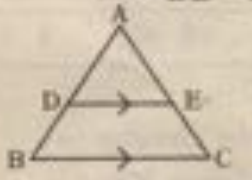
   A) diameter B) height C) radius D) slant height

9. $\Delta ABC \sim \Delta XYZ$, AB : XY = 9 : 16, then ar (ΔABC) : ar (ΔXYZ) is [ ]

   A) 3 : 4 B) 4 : 3 C) 81 : 256 D) 256 : 81

10. In the given figure, DE ∥ BC, if $\frac{AD}{DB} = \frac{3}{2}$ and EC = 3.6 cm, then AE = [ ]

    A) 4.5 cm
    B) 5.6 cm
    C) 5.4 cm
    D) 4.6 cm



SOLUTIONS

PART - A

SECTION – I

1. When a dice is rolled, find the probability of getting odd prime number.

Sol. The probability of getting odd prime number

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

2. Write the formula to find the volume of a cone and explain each term in it.

Sol. Volume of a cone $= \frac{1}{3}\pi r^2 h$

where, r = radius, h = height.

3. Find the volume of liquid that hemispherical bowl can hold, where radius of the bowl is 4.2 cm.

Sol. Radius = r = 4.2 cm

Volume of the liquid in hemispherical bowl $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$$

$$= 44 \times 1.4 \times 0.6 \times 4.2$$

$$= 155.232 \text{ cm}^3$$

4. Prove that

$$\mathbf{4\tan^2 45^\circ - \operatorname{cosec}^2 30^\circ + \cos^2 30^\circ = \frac{3}{4}}.$$

Sol. $4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2$

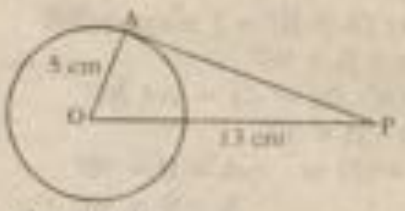
$$= 4(1) - 4 + \frac{3}{4}$$

$$= 4 - 4 + \frac{3}{4} = \frac{3}{4}$$

5. Find the length of the tangent to circle from a point 13 cm away from the centre of a circle of radius 5 cm.

Sol. Radius = OA = 5 cm

OP = 13 cm



From ΔAOP

$$PA = \sqrt{OP^2 - OA^2}$$

$$= \sqrt{13^2 - 5^2}$$

$$= \sqrt{169 - 25} = \sqrt{144} = 12 \text{ cm}$$

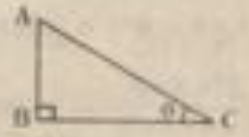
6. Find the mean of prime numbers which are less than 30.

Sol. Mean $= \frac{\text{sum of the observations}}{\text{number of observation}}$

$$= \frac{2+3+5+7+11+13+17+19+23+29}{10}$$

$$= \frac{129}{10} = 12.9$$

7. Using the figure given of ΔABC, prove that $\sin^2\theta + \cos^2\theta = 1$.



Sol. From given figure,

$$\sin\theta = \frac{AB}{AC} \text{ and } \cos\theta = \frac{BC}{AC}$$

$$\text{LHS} = \sin^2\theta + \cos^2\theta$$

$$= \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2$$

$$= \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \frac{AB^2 + BC^2}{AC^2}$$

$$= \frac{AC^2}{AC^2} \quad [\because AB^2 + BC^2 = AC^2]$$

$$= 1 = \text{RHS}$$

Hence it is proved.

SECTION – II

8. If $\operatorname{cosec}(A+B) = 1$ and $\cot(A-B) = \sqrt{3}$, $0^\circ < A + B \le 90^\circ$, $A > B$, then find A and B.

Sol. $\operatorname{cosec}(A+B) = 1 = \operatorname{cosec} 90^\circ$

$A + B = 90^\circ$ ..... (1)

$\cot(A-B) = \sqrt{3} = \cot 30^\circ$

$A - B = 30^\circ$ ..... (2)

(1) + (2) $\Rightarrow$

$$\begin{array}{rl} A + B &= 90^\circ \\ A - B &= 30^\circ \\ \hline 2A &= 120^\circ \end{array}$$

$$\Rightarrow A = \frac{120^\circ}{2} = 60^\circ$$

$A + B = 90^\circ$

$60^\circ + B = 90^\circ \Rightarrow B = 90^\circ - 30^\circ = 30^\circ$

$\therefore A = 60^\circ, B = 30^\circ$

9. The diameter of the base of a right circular cone is 12 cm and volume 376.8 cm^3. Find its height ($\pi = 3.14$).

Sol. Diameter = d = 12 cm

$$r = \frac{d}{2} = \frac{12}{2} \text{ cm} = 6 \text{ cm}$$

Volume of a cone $= \frac{1}{3}\pi r^2 h$

Volume of a cone = 376.8 cm^3

$$\frac{1}{3} \times 3.14 \times 6 \times 6 \times h = 376.8$$

$$h = \frac{376.8 \times 3}{3.14 \times 6 \times 6} = 10 \text{ cm}$$

10. A bag contains 7 red, 5 white and 6 black balls. A ball is drawn from the bag at random, find the probability that the ball drawn is not black.

Sol. Number of total outcomes

$= 7 + 5 + 6 = 18$

Number of favourable outcomes

$= 7 + 5 = 12$

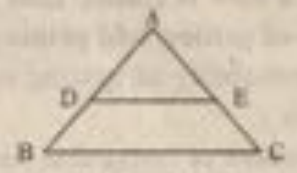
Probability that the ball drawn is not black

$$= \frac{\text{Number of favourable outcomes}}{\text{Number of total possible outcomes}}$$

$$= \frac{12}{18} = \frac{2}{3}$$

11. In ΔABC, D and E are points on AB and AC respectively. If AB = 14 cm, AD = 3.5 cm, AE = 2.5 cm and AC = 10 cm, show that DE $\parallel$ BC.

Sol. Given AB = 14 cm, AD = 3.5 cm, AE = 2.5 cm, AC = 10 cm.



BD = AB – AD = 14 – 3.5 = 10.5 cm

EC = AC – AE = 10 – 2.5 = 7.5 cm

$$\frac{AD}{DB} = \frac{3.5}{10.5} = \frac{1}{3} \quad \text{..... (1)}$$

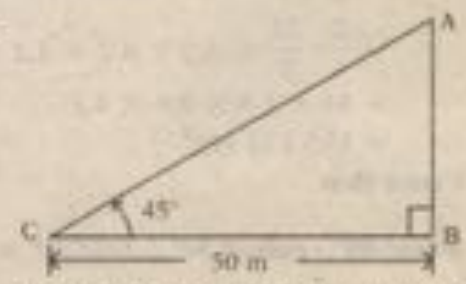
$$\frac{AE}{EC} = \frac{2.5}{7.5} = \frac{1}{3} \quad \text{..... (2)}$$

From (1) and (2), $\frac{AD}{DB} = \frac{AE}{EC}$

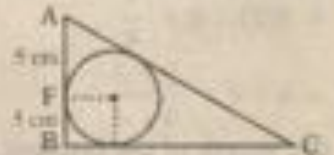
$\therefore$ DE $\parallel$ BC ($\because$ using converse of BPT)

12. The angle of elevation of the top of a tower from a point on the ground, which is 50 m away from the foot of the tower is 45°. Draw the diagram for the situation.

Sol.

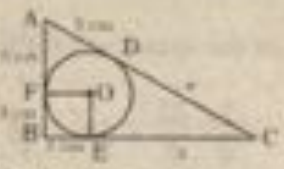


13. A circle of radius 3 cm is inscribed in a triangle ABC and AF = 5 cm, BF = 3 cm as shown in the figure.



Somu said that the measure of the side AC is 17 cm. Do you agree ? Give reasons.

Sol. AB = 8 cm, AD = 5 cm, BE = 3 cm



Let CD = CE = x

AC = 5 + x, BC = 3 + x

Since OE = OF = radius and

OF ⊥ AB, OE ⊥ BC

∴ AB ⊥ BC

From ΔABC

$AC^2 = AB^2 + BC^2$

$(5 + x)^2 = (8)^2 + (3 + x)^2$

$(\because a^2 - b^2 = (a + b)(a - b))$

$(8 + 2x)2 = 64$

$8 + 2x = 32$

$2x = 24 \Rightarrow x = 12$

∴ AC = 5 + x = 5 + 12 = 17 cm

SECTION – III

14. A metallic sphere of diameter 30 cm is melted and recast into a cylinder of radius 10 cm. Find the height of the cylinder.

Sol. Diameter of metallic sphere = d

= 30 cm

$R = \frac{d}{2} = 15$ cm

Radius of a cylinder = r = 10 cm

Volume of a cylinder = Volume of a sphere

$\pi r^2 h = \frac{4}{3}\pi R^3$

$10 \times 10 \times h = \frac{4}{3} \times 15 \times 15 \times 15$

$h = \frac{4 \times 15 \times 15 \times 15}{3 \times 10 \times 10} = 45$ cm

OR

Two boys on either side of their school building of 20 m height observes its top at the angles of elevation 30° and 60° respectively. Find the distance between two boys.

Sol. Let BC = Distance between two boys

AD = Height of the school building

= 20 m

From ΔABD,

$\cot 30° = \frac{BD}{AD} \Rightarrow \sqrt{3} = \frac{BD}{20}$

$BD = 20\sqrt{3}$ m.

From ΔACD,

$\cot 60° = \frac{DC}{AD}$

$\frac{1}{\sqrt{3}} = \frac{DC}{20} \Rightarrow DC = \frac{20}{\sqrt{3}}$ m

∴ BC = BD + DC

$= 20\sqrt{3} + \frac{20}{\sqrt{3}} = \frac{80}{\sqrt{3}}$ m

15. Prove that : $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sec\theta - \tan\theta$

Sol. LHS = $\sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}}$

Multiplying both numerator and denominator with $\sqrt{1 - \sin\theta}$

$= \sqrt{\frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}}$

$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}}$

$= \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}}$ $(\because 1 - \sin^2\theta = \cos^2\theta)$

$= \frac{1 - \sin\theta}{\cos\theta}$

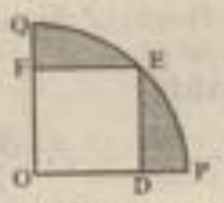
$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$

$= \sec\theta - \tan\theta$ = RHS

Hence it is proved.

OR

A square ODEF is inscribed in a quadrant OPEQ of circle and OD = $14\sqrt{2}$ cm. Aarthi said that "the area of shaded region is224 cm².". Do you agree ? Give reasons.



Sol. Side of the square = OD = $14\sqrt{2}$ cm

Length of the diagonal = OE

$= \sqrt{2}\,(OD)$

$= \sqrt{2}\,(14\sqrt{2}) = 28$ cm

Radius of the quadrant OPEQ = r = diagonal of the square ODEF.

∴ Radius of the quadrant OPEQ = r

= 28 cm

Area of the quadrant OPEQ $= \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28$

$= 22 \times 28 = 616\ cm^2$

Area of the square ODEF = $(OD)^2$

$= (14\sqrt{2})^2 = 196 \times 2 = 392\ cm^2$

∴ The area of the shaded region

= Area of the quadrant OPEQ

– Area of the square ODEF

$= 616 - 392 = 224\ cm^2$

Yes, I agree with Aarthi statement.

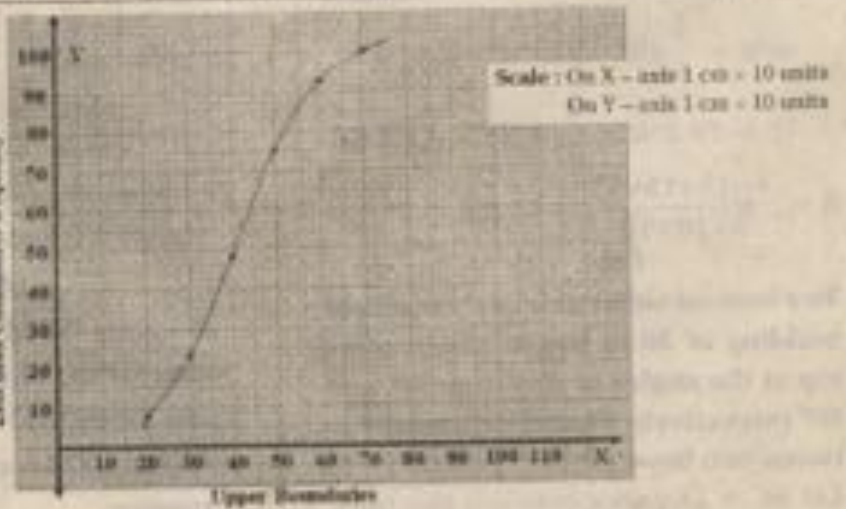
16. The following table shows that ages of the patients admitted in a hospital during a year.

Age in years	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
No. of Patients	8	15	25	27	18	7

Draw a less than Ogive curve for the above data.

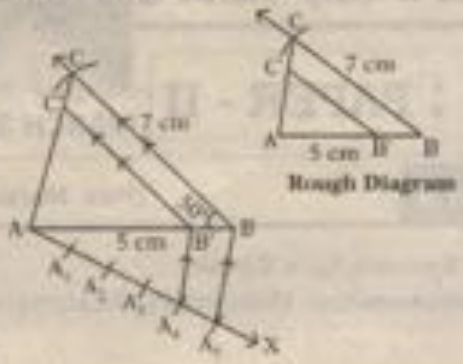
Sol.

Age (in years)	Number of patients	Upper boundaries	Less than cumulative frequency	(x, y)
10-20	8	20	8	(20, 8)
20-30	15	30	23	(30, 23)
30-40	25	40	48	(40, 48)
40-50	27	50	75	(50, 75)
50-60	18	60	93	(60, 93)
60-70	7	70	100	(70, 100)



Construct a triangle ABC in which AB = 5 cm, BC = 7 cm and ∠ABC = 50°, then construct a triangle similar to it, whose sides are 4/5 of the corresponding sides of first triangle.

Sol.



Rough Diagram

Steps of Construction :

1) Draw a triangle ABC with AB = 5 cm, BC = 7 cm and ∠ABC = 50°.
2) Draw a ray $\overrightarrow{AX}$ such that ∠BAX is an acute angle.
3) Draw A_1, A_2, A_3, A_4, A_5 arcs on $\overrightarrow{AX}$ such that $AA_1 = A_1A_2 = \ldots\ldots A_4A_5$.
4) Join A_5 and B.
5) Draw a parallel line to A_5B through A_4 to meet AB at B'.
6) Draw a parallel line to BC through B' to meet AC at C'.
7) ΔAB'C' is required similar triangle.

17. The below distribution gives the weight of 40 students in a class. Find the median weight of the students.

Weight in kg.	30 - 35	35 - 40	40 - 45	45 - 50	50 - 55	55 - 60
No. of students	4	5	10	8	8	5

Sol.

Weight (in kg)	Number of students	L.C.F
30 - 35	4	4
35 - 40	5	9
40 - 45	10	19 (cf)
45 - 50	8 (f)	27 .. median class
50 - 55	8	35
55 - 60	5	40

Here $\frac{n}{2} = \frac{40}{2} = 20$, Median class : 45-50.

$l = 45, f = 8, cf = 19, h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h,$$

$$= 45 + \left(\frac{20 - 19}{8}\right) \times 5$$

$$= 45 + \frac{5}{8} = 45 + 0.625 = 45.625$$

OR

Suppose you drop a dice at random on the circular region of diameter 28 cm as shown in the figure. What is the probability that it will land inside the rectangle ?

Sol. Diameter of circle

$= d = 28$ cm

$r = d/2 = 14$ cm

Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 14 \times 14 = 22 \times 28 = 616 \text{ cm}^2$$

Area of the rectangle $= l \times b$

$$= 11 \times 7 = 77 \text{ cm}^2$$

∴ Probability that it will land inside the rectangle $= \frac{\text{Area of the Rectangle}}{\text{Area of the circle}}$

$$= \frac{77}{616} = \frac{7}{56} = \frac{1}{8}$$

PART - B

1) A 2) C 3) D 4) B 5) C 6) C 7) D 8) B 9) C 10) C