CLASS - X

TELANGANA

A GRADE

MODEL PAPER 5

MATHEMATICS : PAPER - II

JUNE 2017

Time : 2 hours 45 min.] **Parts - A and B** [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. *In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.*
2. *Answer the Questions under 'Part – A' on a separate answer book.*
3. *Write the answers to the Questions under 'Part – B' on the Question paper itself and attach it to the answer book of 'Part – A'.*

Time : 2 Hours] **PART - A** [Marks : 35

Note :

1. Answer **all** the questions from the given **three** sections –I, II and III of **Part – A**.
2. In section – III, every question has internal choice. Answer **anyone** alternative.

SECTION – I

(Marks : 7 × 1 = 7)

Note : (i) Answer **all** the questions.

(ii) Each question carries **1** mark.

1. If a cone is inscribed in a cylinder, what is the ratio of their volumes ?
2. The vertex angle of a cone is 60°. Find the ratio of diameter with the height of the cone.
3. Is it correct to say that $\sin\theta = \cos(90 - \theta)$? Why ?
4. Draw the diagram corresponding to Basic Proportionality Theorem.
5. "Cuboid is one of right prism". Is it true ? Justify.
6. A dice is thrown once. Find the probability of getting a composite number.
7. Write the first 10 prime numbers and find their median.

SECTION – II

(Marks : 6 × 2 = 12)

Note : (i) Answer **all** the questions.

(ii) Each question carries **2** marks.

8. If A, B, C are interior angles of ΔABC, then show that

$$\sin\left(\frac{A+B}{2}\right) + \cos\left(\frac{A+B}{2}\right) = \cos\frac{C}{2} + \sin\frac{C}{2}.$$

9. From the following data, find the probability of selecting 'B' blood group student.

Blood group	A	B	AB	O
Number of students	10	13	12	5

10. A straight highway leads to foot of the tower. A man standing at the top of the tower observes a car at an angle of depression of θ, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression is ϕ. Draw a diagram for this data and analyze.

11. Prove that $\sqrt{\frac{\text{cosec } A + 1}{\text{cosec } A - 1}} - \sqrt{\frac{\text{cosec } A - 1}{\text{cosec } A + 1}} = 2 \cot A$.

12. The height of 12 members of a family are given below in the table.

Height (in ft)	5	5.2	5.4	5.6
No.of family members	3	4	3	2

Find the mean height of the family members.

13. What can you say about the values of sin A and cos A as the measure of an angle A increases from 0° to 90° ?

SECTION – III

(Marks : 4 × 4 = 16)

Note : (i) Answer **all** the questions.

(ii) Each question carries **four** marks.

(iii) There is internal choice for each question, only one option from each question is to be attempted.

14. Ten identical mementos is made by a school to awarding 10 students for first prize winners in games. If each memento is made as shown in figure (shaded portion) its base PQRS is silver plated from the front side at the rate of ₹ 20 per square cm. Find the total cost of the silver plating of 10 mementos. (OR = 5 cm, RQ = 6 cm, PS = 8 cm)

OR

If $\frac{\cos\alpha}{\cos\beta} = m$, $\frac{\cos\alpha}{\sin\beta} = n$, then show that $(m^2 + n^2)\cos^2\alpha = m^2n^2$.

15. Draw a two concentric circles of radii 1.5 cm and 4 cm. From a point 10 cm away from its centre. Construct the pairs of tangent to the circles.

OR

Daily income of 40 coal-mine labours are given in the following table :

Daily income in Rupees	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350	350 – 400
No.of labours	4	3	3	8	13	9

Draw Ogive curves (Cumulative frequency) for this data.

16. The angle of elevation of the top of a tower from two points at a distance of 4m and 9 m from the base of the tower and in the same straight line with it, are complementary. Prove that the height of the tower is 6 m.

OR

Find the missing frequencies f_1 and f_2, if mean of 50 observations given below is 36.4.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	3	5	f_1	10	f_2	8	5

17. If $\text{cosec}\,\theta + \cot\theta = k$, then write all trigonometric ratios at θ in terms of k.

OR

ABCD is a trapezium, in which AB || DC and its diagonals intersect each other at point 'O'. Show that $\frac{OA}{OB} = \frac{OC}{OD}$.

Time : 30 Mts.] **PART - B** **[Marks : 5**

Instructions :

(i) Answer **all** questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in question paper only.

(iv) Marks will not be awarded in any case of over - writing, rewriting or erased answers.

1. Write the **CAPITAL LETTERS** showing the correct answer for the following questions in the brackets provided against each question. **(Marks 10 × ½ = 5)**

1. **Base area of the prism is 30 cm² and its height is 10 cm. Then the volume of the prism is .....** [ ]

(A) 300 cm³ (B) 300 cm² (C) 150 cm² (D) 150 cm³

2. **If ΔPQR ~ ΔXYZ, QR = 3 cm, YZ = 4 cm, ΔPQR area = 54 cm².**

Then ΔXYZ area = ... [ ]

(A) 13.5 cm² (B) 46 cm² (C) 96 cm² (D) 12 cm²

3. Which one of the following is NOT a measure of central tendency ? [ ]

(A) Mean (B) Median (C) Range (D) Mode

4. In ΔABC with $\angle A = 90°$; from A, perpendicular AD is drawn on BC. Which one of the following is NOT correct ? [ ]

(A) $\Delta ABC \sim \Delta DAC$ (B) $\Delta DAC \sim \Delta DBA$

(C) $\Delta ABC \sim \Delta DBA$ (D) $\Delta ABC \sim \Delta DBA \sim \Delta DAC$

5. At what value of 'x', $\frac{5}{x}$ may possible probability of an event ? [ ]

(A) 2 (B) 1 (C) 4 (D) 6

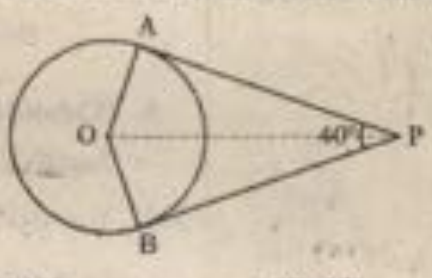
6. Which one of the following is NOT defined ? [ ]

(A) $\sin 90°$ (B) $\cos 0°$ (C) $\sec 90°$ (D) $\cos 90°$

7. $\sqrt{\frac{1-\cos^2 A}{1+\cot^2 A}}$ = ... [ ]

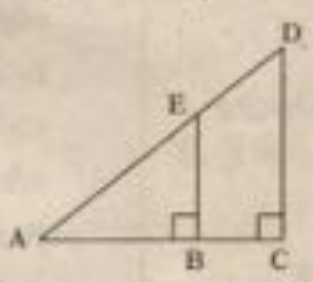
(A) $\sin A$ (B) $\sqrt{\sin A}$ (C) $\sin^2 A$ (D) $\sin^4 A$

8. From the below figure $\angle APB = 40°$, then $\angle AOB$ = ... [ ]



(A) 110° (B) 140° (C) 80° (D) 160°

9. In the below figure, AB = 3 cm, AC = 8 cm, BE = 4.5 cm, then CD = ... [ ]



(A) 10.5 cm (B) 9.5 cm (C) 16 cm (D) 12 cm

10. The angle of depression from the top of a tower 12 m height, at a point on the ground is 30°. The distance of the point from the top is ..... [ ]

(A) 10 m (B) $12\sqrt{3}$ m (C) 7.5 m (D) 6 m

✣ ✣ ✣ ✣ ✣

SOLUTIONS

PART - A

SECTION – I

1. **If a cone is inscribed in a cylinder, what is the ratio of their volumes ?**

Sol. Volume of the cone :

Volume of the cylinder

$= \frac{1}{3}\pi r^2 h : \pi r^2 h$

$= \frac{1}{3} : 1$

$= 1 : 3$

2. **The vertex angle of a cone is 60°. Find the ratio of diameter with the height of the cone.**

Sol. $\tan 30° = \frac{r}{h}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{d/2}{h} \left(\because r = \frac{d}{2}\right)$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{d}{2h}$

$\Rightarrow \frac{2}{\sqrt{3}} = \frac{d}{h}$

i.e., Ratio of diameter with height $= 2 : \sqrt{3}$

3. **Is it correct to say that $\sin\theta = \cos(90 - \theta)$? Why ?**

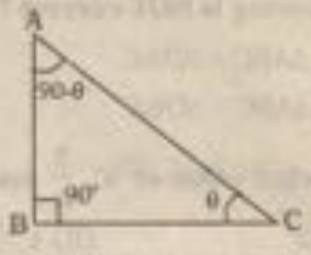
Sol. Yes.

$\because \sin\theta = \frac{AB}{AC} \left(\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}\right)$

$\cos(90 - \theta)$

$= \frac{AB}{AC} \left(\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}}\right)$

$\therefore \sin\theta = \cos(90 - \theta)$



4. **Draw the diagram corresponding to Basic Proportionality Theorem.**

Sol.

5. **"Cuboid is one of right prism". Is it true ? Justify.**

Sol. Yes. Because

In a prism if lateral surfaces are at right angles to the bases along the edges then it is called a Right Prism.

In a cuboid the lateral surfaces are at right angles to the base along the edges.

Hence cuboid is a right prism.

6. **A dice is thrown once. Find the probability of getting a composite number.**

Sol. Possible outcomes = 1, 2, 3, 4, 5, 6

Favourable outcomes = 4, 6

Probability of a getting a compositive number

$$= \frac{\text{Number of favourable outcomes}}{\text{Total possible outcomes}}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

7. Write the first 10 prime numbers and find their median.

Sol. The first 10 prime numbers are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Median of the above data $= \frac{11+13}{2}$

$$= \frac{24}{2} = 12$$

SECTION – II

8. If A, B, C are interior angles of ΔABC, then show that

$$\sin\left(\frac{A+B}{2}\right) + \cos\left(\frac{A+B}{2}\right) = \cos\frac{C}{2} + \sin\frac{C}{2}.$$

Sol. $A + B + C = 180°$

$A + B = 180° - C$

$$\left(\frac{A+B}{2}\right) = 90° - \frac{C}{2}$$

$$\sin\left(\frac{A+B}{2}\right) = \sin\left(90° - \frac{C}{2}\right)$$

$$= \cos\frac{C}{2} \quad \ldots\ldots\ldots (1)$$

$$\cos\left(\frac{A+B}{2}\right) = \cos\left(90° - \frac{C}{2}\right)$$

$$= \sin\frac{C}{2} \quad \ldots\ldots\ldots (2)$$

(1) + (2)

$$= \sin\left(\frac{A+B}{2}\right) + \cos\left(\frac{A+B}{2}\right)$$

$$= \cos\frac{C}{2} + \sin\frac{C}{2}$$

9. From the following data, find the probability fo selecting 'B' blood group student.

Blood group	A	B	AB	O
Number of students	10	13	12	5

Sol. Number of favourable outcomes = 13

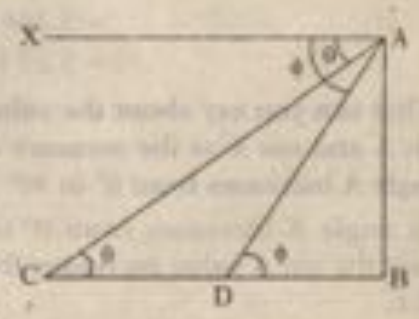
Number of total possible outcomes = 40

∴ Propability of selecting B blood group $= \frac{\text{No.of Favourable outcomes}}{\text{No. of Total outcomes}}$

$$= \frac{13}{40}$$

10. A straight highway leads to foot of the tower. A man standing at the top of the tower observes a car at an angle of depression of θ, which is approaching the foot of the tower with a uniform speed. Six seconds later the angle of depression is ϕ. Draw a diagram for this data and analyze.

Sol.



OR

If $\frac{\cos\alpha}{\cos\beta} = m$, $\frac{\cos\alpha}{\sin\beta} = n$, then show that $(m^2 + n^2)\cos^2\alpha = m^2n^2$.

Sol. $m = \frac{\cos\alpha}{\cos\beta}$ $\quad n = \frac{\cos\alpha}{\sin\beta}$

$\frac{1}{m} = \frac{\cos\beta}{\cos\alpha}$ $\quad \frac{1}{n} = \frac{\sin\beta}{\cos\alpha}$

$\frac{1}{m^2} = \frac{\cos^2\beta}{\cos^2\alpha}$ $\quad \frac{1}{n^2} = \frac{\sin^2\beta}{\cos^2\alpha}$

$\frac{1}{m^2} + \frac{1}{n^2} = \frac{\cos^2\beta}{\cos^2\alpha} + \frac{\sin^2\beta}{\cos^2\alpha}$

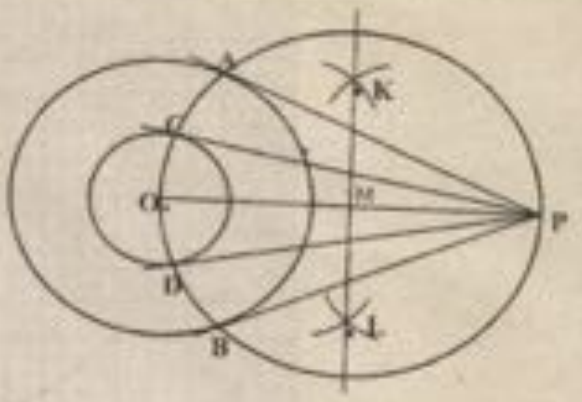
$\frac{m^2 + n^2}{m^2n^2} = \frac{\sin^2\beta + \cos^2\beta}{\cos^2\alpha}$

$(m^2 + n^2)\cos^2\alpha = m^2n^2$

15. Draw a two concentric circles of radii 1.5 cm and 4 cm. From a point 10 cm away from its centre. Construct the pairs of tangent to the circles.

Sol.

Rough Diagram



Steps of constructions :

1) Draw concentric circles with centre "O" and radii 1.5 cm and 4 cm.
2) Mark an external point P such that OP = 10 cm
3) Draw the perpendicular bisector of OP which meets it at M.
4) Draw a circle with centre M having radius OM, to intersects the concentric circle at A, B, C and D respectively.
5) Join P, A; P, B; P, C; and P, D.

PA, PB, PC, PD are required tangents.

OR

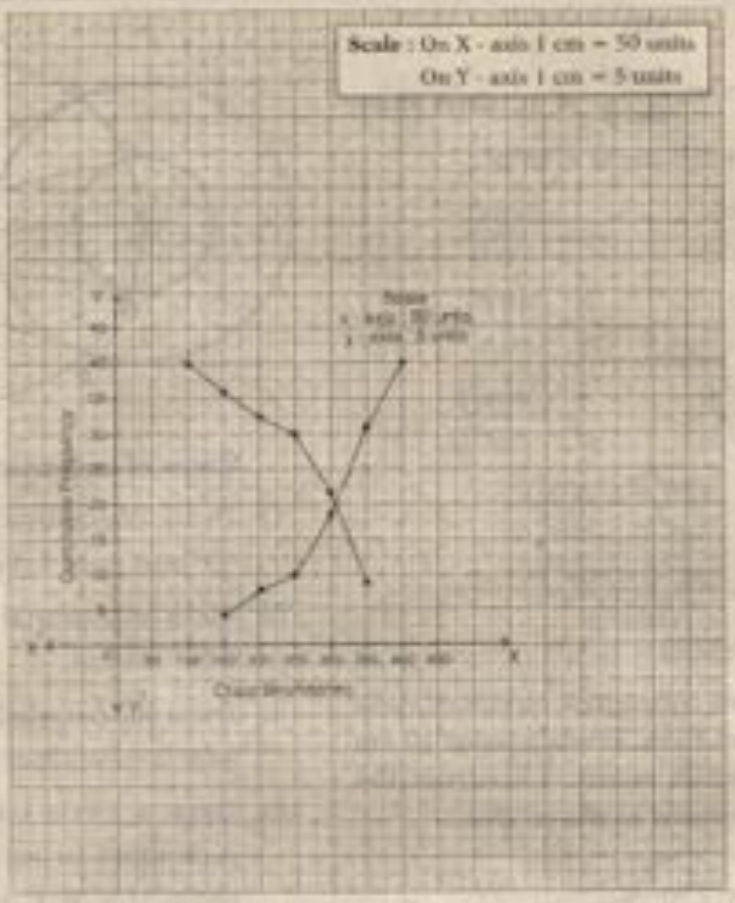
Daily income of 40 coal-mine labours are given in the following table :

Daily income in Rupees	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350	350 - 400
No.of labours	4	3	3	8	13	9

Draw Ogive curves (Cumulative frequency) for this data.

Sol.

Daily income in Rs	No. of labours	U.B	Less than cumulative frequency	(x, y)	L.B.	More than cumulative frequency	(x, y)
100 – 150	4	150	4	(150, 4)	100	40	(100, 40)
150 – 200	3	200	7	(200, 7)	150	36	(150, 36)
200 – 250	3	250	10	(250,10)	200	33	(200, 33)
250 – 300	8	300	18	(300, 18)	250	30	(250, 30)
300 – 350	13	350	31	(350, 31)	300	22	(300, 22)
350 – 400	9	400	40	(400, 40)	350	9	(350, 9)



16. **The angle of elevation of the top of a tower from two points at a distance of 4m and 9 m from the base of the tower and in the same straight line with it, are complementary. Prove that the height of the tower is 6 m.**

Sol. $\tan\theta = \frac{h}{9}$ ——— (1)

$\tan(90-\theta) = \frac{h}{4}$

$\cot\theta = \frac{h}{4} \Rightarrow \tan\theta = \frac{4}{h}$ ——— (2)

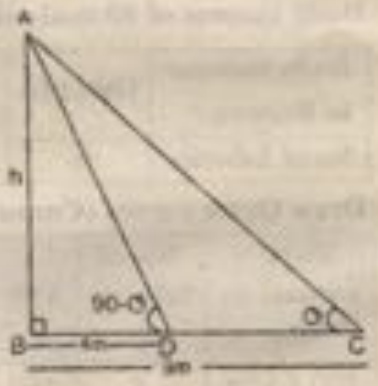
from (1) and (2)

$\frac{h}{9} = \frac{4}{h}$

$h^2 = 36$

$h = 6$

$\therefore$ Height of the tower = 6m



OR

OR

If $\frac{\cos\alpha}{\cos\beta} = m$, $\frac{\cos\alpha}{\sin\beta} = n$, then show that $(m^2 + n^2)\cos^2\alpha = m^2n^2$.

Sol. $m = \frac{\cos\alpha}{\cos\beta}$ $\quad n = \frac{\cos\alpha}{\sin\beta}$

$\frac{1}{m} = \frac{\cos\beta}{\cos\alpha}$ $\quad \frac{1}{n} = \frac{\sin\beta}{\cos\alpha}$

$\frac{1}{m^2} = \frac{\cos^2\beta}{\cos^2\alpha}$ $\quad \frac{1}{n^2} = \frac{\sin^2\beta}{\cos^2\alpha}$

$\frac{1}{m^2} + \frac{1}{n^2} = \frac{\cos^2\beta}{\cos^2\alpha} + \frac{\sin^2\beta}{\cos^2\alpha}$

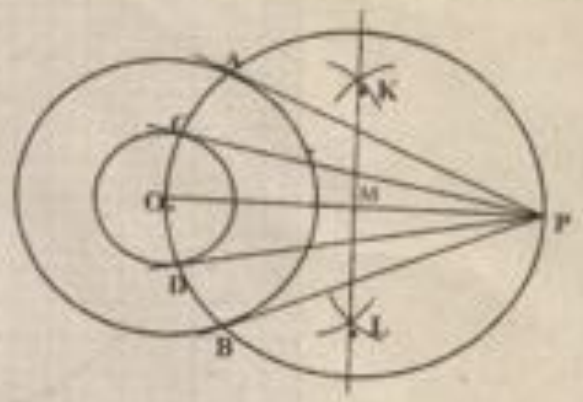
$\frac{m^2 + n^2}{m^2n^2} = \frac{\sin^2\beta + \cos^2\beta}{\cos^2\alpha}$

$(m^2 + n^2)\cos^2\alpha = m^2n^2$

15. Draw a two concentric circles of radii 1.5 cm and 4 cm. From a point 10 cm away from its centre. Construct the pairs of tangent to the circles.

Sol.

Rough Diagram



Steps of constructions :

1) Draw concentric circles with centre "O" and radii 1.5 cm and 4 cm.
2) Mark an external point P such that OP = 10 cm
3) Draw the perpendicular bisector of OP which meets it at M.
4) Draw a circle with centre M having radius OM, to intersects the concentric circle at A, B, C and D respectively.
5) Join P, A; P, B; P, C; and P, D.

PA, PB, PC, PD are required tangents.

OR

Daily income of 40 coal-mine labours are given in the following table :

Daily income in Rupees	100 - 150	150 - 200	200 - 250	250 - 300	300 - 350	350 - 400
No.of labours	4	3	3	8	13	9

Draw Ogive curves (Cumulative frequency) for this data.

Sol.

Daily income in Rs	No. of labours	U.B	Less than cumulative frequency	(x, y)	L.B.	More than cumulative frequency	(x, y)
100 – 150	4	150	4	(150, 4)	100	40	(100, 40)
150 – 200	3	200	7	(200, 7)	150	36	(150, 36)
200 – 250	3	250	10	(250,10)	200	33	(200, 33)
250 – 300	8	300	18	(300, 18)	250	30	(250, 30)
300 – 350	13	350	31	(350, 31)	300	22	(300, 22)
350 – 400	9	400	40	(400, 40)	350	9	(350, 9)

Find the missing frequencies f_1 and f_2, if mean of 50 observations given below is 36.4.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	3	5	f_1	10	f_2	8	5

Sol.

Class CI	Frequency (f)	Mid values x	fx
0 – 10	3	5	15
10 – 20	5	15	75
20 – 30	f_1	25	$25 f_1$
30 – 40	10	35	350
40 – 50	f_2	45	$45 f_2$
50 – 60	8	55	440
60 – 70	5	65	325

$\Sigma f = 31 + f_1 + f_2$ $\quad\quad$ $\Sigma fx = 1205 + 25 f_1 + 45 f_2$

$\text{Mean} = \dfrac{\Sigma f_x}{\Sigma f}$

$36.4 = \dfrac{1205 + 25f_1 + 45f_2}{31 + f_1 + f_2}$ $\quad\quad$ $31 + f_1 + f_2 = 50$

$36.4 = \dfrac{1205 + 25f_1 + 45f_2}{50}$ $\quad\quad$ $f_1 + f_2 = 50 - 31$

$1820 = 1205 + 25f_1 + 45f_2$ $\quad\quad$ $= 19$

$1820 - 1205 = 25(f_1 + f_2) + 20 f_2$

$615 - 475 = 20f_2$

$140 = 20f_2$

$f_2 = 7$

$f_1 = 19 - 7 = 12$

17. If $\operatorname{cosec}\theta + \cot\theta = k$, then write all trigonometric ratios at θ in terms of k.

Sol. $\operatorname{cosec}\theta + \cot\theta = k$ $\quad$ — (1)

$\operatorname{cosec}\theta - \cot\theta = \dfrac{1}{k}$ $\quad$ — (2)

$(1) + (2) \Rightarrow 2\operatorname{cosec}\theta = k + \dfrac{1}{k}$

$\operatorname{cosec}\theta = \dfrac{k^2+1}{2k} \Rightarrow \sin\theta = \dfrac{2k}{k^2+1}$

$$\cos\theta = \sqrt{1-\frac{4k^2}{(k^2+1)^2}} = \sqrt{\frac{(k^2-1)^2}{(k^2+1)^2}} = \frac{k^2-1}{k^2+1}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{2k}{k^2+1}\right)}{\left(\frac{k^2-1}{k^2+1}\right)} = \frac{2k}{k^2-1}$$

$$\cot\theta = \frac{k^2-1}{2k}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{k^2+1}{k^2-1}$$

OR

ABCD is a trapezium, in which AB ∥ DC and its diagonals intersect each other at point 'O'. Show that $\frac{OA}{OB} = \frac{OC}{OD}$.

In Δ OAB, Δ OCD

∠ AOB = ∠ COD (∵ pair of vertically Opp. Angles)

∠ OAB = ∠ OCD (∵ Pair of alternate angles)

∠ OBA = ∠ ODC (∵ Pair of alternate angles)

∴ Δ OAB ~ Δ OCD (∵ AAA similarity)

$$\therefore \frac{OA}{OC} = \frac{OB}{OD}$$

$$\Rightarrow \frac{OA}{OB} = \frac{OC}{OD}$$

PART - B

1. A 2. C 3. C 4. Add score 5. D 6. C 7. C 8. B 9. D