CLASS - X

TELANGANA

A GRADE

MODEL PAPER 7

MATHEMATICS : PAPER - II

JUNE 2016

Time : 2.45 Hours] Parts - A and B [Max. Marks : 40

Similar Triangles, Tangents and Secants to a Circle, Mensuration, Trigonometry, Applications of Trigonometry, Probability, Statistics

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.
2. Answer the Questions under **Part – A** on a separate answer book.
3. Write the answer to the questions under **Part – B** on the question paper itself and attach it to the answer book of **Part – A**.

Time : 2.15 Hours] **PART - A** **[Max. Marks : 35**

Note :

1. Answer **all** the questions from the given **three** sections I, II and III of **Part – A**.
2. In section – III, every question has internal choice. Answer **anyone** alternative.

SECTION – I

$7 \times 1 = 7$

Note : (i) Answer **all** the following questions.

(ii) Each question carries **1** Mark.

1. Find the curved surface area of a cylinder of radius 14 cm and height 21 cm. $\left(\pi = \frac{22}{7}\right)$

2. It is given that $\Delta ABC \sim \Delta DEF$. Is it true to say that $\frac{BC}{DE} = \frac{AB}{EF}$? Justify your answer.

3. Find the probability of getting a prime number, when a card drawn at random from the numbered cards from 1 to 25.

4. Evaluate : $\frac{\sin 58^\circ}{\cos 32^\circ} + \frac{\tan 42^\circ}{\cot 48^\circ}$

5. "The length of the tangent from an external point 'P' to a circle with centre 'O' is always less than OP." Is this statement true ? Give reasons.

6. Write the formula to find the mean of a grouped data, using assumed mean method and explain each term.

7. If a tower of height 'h' is observed from a point with a distance 'd' and angle 'θ', then express the relation among h, d and θ.

SECTION – II

6 × 2 = 12

Note : (i) Answer **all** the following questions.

(ii) Each question carries **2** Marks.

8. If $x = a \sec\theta$ and $y = b \tan\theta$, then prove that $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

9. There are 5 red balls, 4 green balls and 6 yellow balls in a box. If a ball is selected at random, what is the probability of not getting a yellow ball ?

10.

C.I	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency(f_i)	5	8	10	5	2

Find the value of $\sum f_i x_i$ for the above data, where x_i is the mid value of each class.

11. A toy is in the form of a cone mounted on a hemisphere. The radius of the base and the height of the cone are 7 cm and 8 cm respectively. Find the surface area of the toy.

$\left(\pi = \frac{22}{7}\right)$

12. AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 60°$. If AT is the tangent to the circle at the point A, then find the measure of $\angle BAT$.

13. Draw a circle with 5 cm radius and construct a pair of tangents to the circle.

SECTION – III

4 × 4 = 16

Note : (i) Answer **all** the following questions.

(ii) In this section, every question has internal choice.

(iii) Answer **anyone** alternative.

(iv) Each question carries **4** Marks.

14. a) Two dice are rolled at same time and the sum of the numbers appearing on them is noted. Find the probability of getting each sum from 3 to 5 separately.

OR

b) If $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 4$ $(0° < \theta < 90°)$, then find the value of θ.

15. a) The scores of 20 students in a test is tabulated as follows :

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	1	6	7	4	2

Find the mode of the data.

OR

b) Two concentric circles of radii 10 cm and 6 cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.

16. a) A tree is broken without separating from the stem by the wind. The top touches the ground making an angle 30° at a distance of 12 m from the foot of the tree. Find the height of the tree before breaking.

OR

b) How many spherical balls each 7 cm in diameter can be made out of a solid lead cube whose edge measures 66 cm ?

17. a) The literacy rate (in percentage) of 35 cities is given in the following table.

Literacy rate %	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of cities	3	11	10	8	3

Prepare 'more than type' cumulative frequency table and draw ogive curve for this data.

OR

b) Construct a triangle of sides 5 cm, 6 cm and 7 cm. Then construct a triangle similar to it, whose sides are 1½ times the corresponding sides of the first triangle.

Time : 30 Mts.] **PART - B** **[Max. Marks : 5**

Instructions :

(i) Answer **all** the questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in question paper only.

(iv) Marks will **not** be awarded in any case of over - writing, rewriting or erased answers.

I. Write the **CAPITAL LETTERS (A,B,C,D)** showing the correct answer for the following questions in the brackets provided against them. **10 × ½ = 5**

1. **The value of tan θ in terms of cosec θ is .....** [ ]

(A) $\frac{1}{\sqrt{\text{cosec}^2\theta - 1}}$ (B) $\frac{\text{cosec}\,\theta}{\sqrt{\text{cosec}^2\theta - 1}}$ (C) $\frac{2\,\text{cosec}\,\theta}{\sqrt{\text{cosec}^2\theta - 1}}$ (D) $\frac{2}{\sqrt{\text{cosec}^2\theta - 1}}$

2. **Observe the following :** [ ]

(I) $\sin^2 20° + \sin^2 70° = 1$ **(II) $\log_2 (\sin 90°) = 1$**

Which one is CORRECT ?

(A) (I) only (B) (II) only

(C) Both (I) and (II) (D) Neither (I) nor (II)

3. In ΔABC, AC = 12 cm, AB = 5 cm and $\angle BAC = 30°$, the area of ΔABC is ..... [ ]

(A) 30 cm^2 (B) 15 cm^2 (C) 60 cm^2 (D) 20 cm^2

4. Which one of the following can not be the probability of an event ? [ ]

(A) $\frac{2}{3}$ (B) $\frac{4}{5}$ (C) 0.7 (D) $\frac{5}{4}$

5. The x - coordinate of the point of intersection of the two ogives of grouped data is ..... [ ]

(A) median of the data (B) mode of the data

(C) mean of the data (D) average of mid values of the data

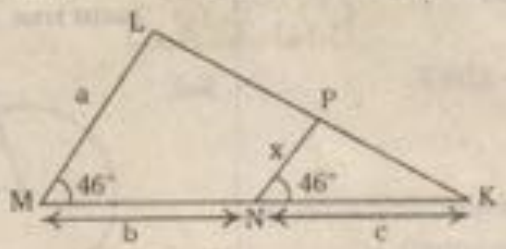
6. Volumes of two spheres are in the ratio of 8 : 27, the ratio of their surface areas is ..... [ ]

(A) 2 : 3 (B) 4 : 3 (C) 2 : 9 (D) 4 : 9

7. A solid ball is exactly fitted inside the cubical box of side 'a'. The volume of the ball is ..... [ ]

(A) $\frac{1}{3}\pi a^3$ (B) $\frac{1}{6}\pi a^3$ (C) $\frac{4}{3}\pi a^3$ (D) $\frac{8}{3}\pi a^3$

8. Express 'x' in terms of a, b and c in the following figure. [ ]



(A) $x = \frac{ac}{b+c}$ (B) $x = \frac{bc}{b+c}$ (C) $x = \frac{b+c}{ac}$ (D) $x = \frac{ab}{a+c}$

9. If the angle of elevation of sun increases from 0° to 90°, then the length of shadow of the tower ..... [ ]

(A) no change (B) increases

(C) decreases (D) can't be decided

10. In a right angled triangle with integral sides at least one of its measurements must be ..... [ ]

(A) multiple of 3 (B) multiple of 9

(C) multiple of 2 (D) multiple of 7

PART - A

SECTION - I

1. Find the curved surface area of a cylinder of radius 14 cm and height 21 cm. $\left(\pi = \frac{22}{7}\right)$

Sol. Given that r = 14 cm, h = 21 cm,

$$\pi = \left(\frac{22}{7}\right)$$

Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 21$$

$$= 1848 \text{ sq.cm}$$

2. It is given that $\Delta ABC \sim \Delta DEF$. Is it true to say that $\frac{BC}{DE} = \frac{AB}{EF}$? Justify your answer.

Sol. Given that $\Delta ABC \sim \Delta DEF$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

($\because$ Ratio of corresponding sides of similar triangles are equal)

But $\frac{BC}{DE} \neq \frac{AB}{EF}$ (given)

$\therefore$ Given statement is wrong.

3. Find the probability of getting a prime number, when a card drawn at random from the numbered cards from 1 to 25.

Sol. Favourable outcomes of prime numbers from 1 to 25 = 9

Total number of outcomes = 25

Probability of getting a prime number

$$\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{25}$$

4. Evaluate : $\frac{\sin 58^\circ}{\cos 32^\circ} + \frac{\tan 42^\circ}{\cot 48^\circ}$.

Sol. $58^\circ, 32^\circ$ and $42^\circ, 48^\circ$ are complementary angles.

$$\frac{\sin 58^\circ}{\cos 32^\circ} + \frac{\tan 42^\circ}{\cot 48^\circ}$$

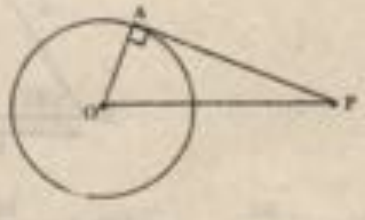
$$= \frac{\sin 58^\circ}{\cos (90 - 58)^\circ} + \frac{\tan 42^\circ}{\cot (90 - 42)^\circ}$$

$$= \frac{\sin 58^\circ}{\sin 58^\circ} + \frac{\tan 42^\circ}{\tan 42^\circ}$$

$$= 1 + 1 = 2$$

5. "The length of the tangent from an external point 'P' to a circle with centre 'O' is always less than OP." Is this statement true ? Give reasons.

Sol.



ΔOAP is right triangle.

OP is hypotenuse. AP is tangent to the circle at A.

$\therefore$ OP > AP

($\because$ Hypotenuse is longest side)

OP > length of the tangent

$\therefore$ Given statement is true.

6. Write the formula to find the mean of a grouped data, using assumed mean method and explain each term.

Sol. Mean = $a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

a – assumed mean

f – frequency

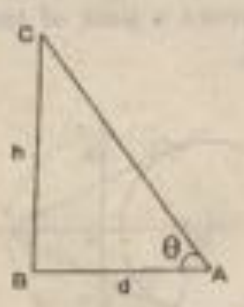
d – x – a

x – class mark

7. If a tower of height 'h' is observed from a point with a distance 'd' and angle 'θ', then express the relation among h, d and θ.

Sol. For writing relation between h, θ, d as

$$\tan\theta = \frac{h}{d}$$



$$\tan\theta = \frac{\text{opposite side of } \theta}{\text{adjacent side of } \theta} = \frac{h}{d}$$

SECTION – II

8. If $x = a \sec\theta$ and $y = b \tan\theta$, then prove that $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Sol. $\frac{x}{a} = \sec\theta, \frac{y}{b} = \tan\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = 1$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

9. There are 5 red balls, 4 green balls and 6 yellow balls in a box. If a ball is selected at random, what is the probability of not getting a yellow ball ?

Sol. Total number of balls in a bag = 15

Total number of chance to select a ball from a bag = 15

Favourable outcomes to select not yellow ball = 9

Probability of not getting a yellow ball

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{15} = \frac{3}{5}$$

10.

C.I	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency(f_i)	5	8	10	5	2

Find the value of $\sum f_i x_i$ for the above data, where x_i is the mid value of each class.

Sol.

C.I	Frequency f_i	Class mark x_i	$f_i x_i$
10 – 20	5	15	75
20 – 30	8	25	200
30 – 40	10	35	350
40 – 50	5	45	225
50 – 60	2	55	110

$\Sigma f_i x_i = 960$

11. **A toy is in the form of a cone mounted on a hemisphere. The radius of the base and the height of the cone are 7 cm and 8 cm respectively. Find the surface area of the toy.** $\left(\pi = \frac{22}{7}\right)$

Sol. According to the data

radius of hemisphere = radius of the base of cone = r

r = 7 cm

height of cone = h = 8 cm

slant height of cone = $l = \sqrt{r^2 + h^2}$

$= \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$

$= \sqrt{113}$

surface area of toy

= curved surface area of cone + surface area of hemisphere

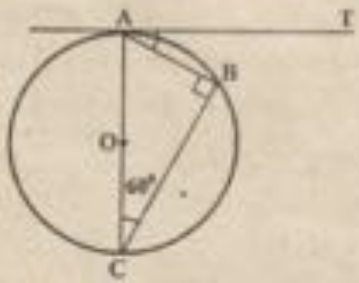
$= \pi rl + 2\pi r^2$

$= \frac{22}{7} \times 7 \times \sqrt{113} + 2 \times \frac{22}{7} \times 7^2$

$= 22\sqrt{113} + 308$ sq.cm

12. **AB is a chord of the circle and AOC is its diameter, such that $\angle ACB = 60°$. If AT is the tangent to the circle at the point A, then find the measure of $\angle BAT$.**

Sol. According to the data $\angle ACB = 60°$



AOC = diameter

AB = chord

AT is a tangent to the circle at A.

$\therefore \angle ACB = 90°$ (∵ semi-circle angle)

$\angle BAC + \angle ACB = 90°$

$\angle BAC + 60° = 90° \Rightarrow \angle BAC = 30°$

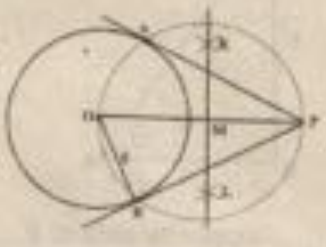
$AT \perp AOC$, $\angle CAT = 90°$

$\angle BAC + \angle BAT = 90°$

$30° + \angle BAT = 90° \Rightarrow \angle BAT = 60°$

13. **Draw a circle with 5 cm radius and construct a pair of tangents to the circle.**

Sol.



SECTION – III

14.a) **Two dice are rolled at same time and the sum of the numbers appearing on them is noted. Find the probability of getting each sum from 3 to 5 separately.**

Sol. Total number of possible outcomes when rolling two dice = 6 × 6 = 36

Favourable outcomes of getting each sum from 3 to 6 separately :

Outcomes of

i) getting sum 3 : (1, 2) (2, 1)

ii) getting sum 4 : (1, 3) (2, 2) (3, 1)

iii) getting sum 5 : (1, 4) (2, 3) (3, 2) (4, 1)

Total number of favourable outcomes

= 9

Probability of getting each sum from 3 to 5 separately

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$= \frac{9}{36} = \frac{1}{4}$$

OR

b) If $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 4$ $(0^\circ < \theta < 90^\circ)$, then find the value of θ.

Sol. $\frac{\sin\theta}{1-\cos\theta} + \frac{\sin\theta}{1+\cos\theta} = 4 \quad (0^\circ < \theta < 90^\circ)$

$$\frac{\sin\theta(1+\cos\theta) + \sin\theta(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)} = 4$$

$$\frac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{1-\cos^2\theta} = 4$$

$$\frac{2\sin\theta}{\sin^2\theta} = 4 \qquad (\because 1-\cos^2\theta = \sin^2\theta)$$

$$\frac{2}{\sin\theta} = 4$$

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30^\circ$$

$$\theta = 30^\circ$$

15. a) The scores of 20 students in a test is tabulated as follows :

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	1	6	7	4	2

Find the mode of the data.

Sol.

Values	Number of Students	
10 – 20	1	
20 – 30	6	
30 – 40	7	← Modal Class
40 – 50	4	
50 – 60	2	

$l = 30, f_0 = 6, f_1 = 7, f_2 = 4, h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{7-6}{2(7)-6-4}\right) \times 10$$

$$= 30 + \left(\frac{1}{14-10}\right) \times 10$$

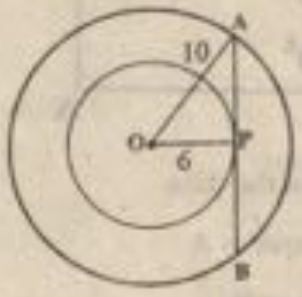
$$= 30 + \left(\frac{1}{4}\right) \times 10$$

$$= 30 + 2.5 = 32.5$$

OR

b) Two concentric circles of radii 10 cm and 6 cm are drawn. Find the length of the chord of the larger circle which touches the smaller circle.

Sol.



Radius of outer circle = R = 10 cm

Radius of inner circle = r = 6 cm

Given that AB in a chord of outer circle and tangent to inner circle of P.

Draw OP ⊥ AB

Then PA = PB

In △ OAP,

$\angle OPA = 90^\circ$

$OP^2 + PA^2 = OA^2$

$6^2 + PA^2 = 10^2$

$PA^2 = 100 - 36 = 64$

$PA = 8$ cm

$\therefore AB = PA + PB = 2PA = 2(8)$

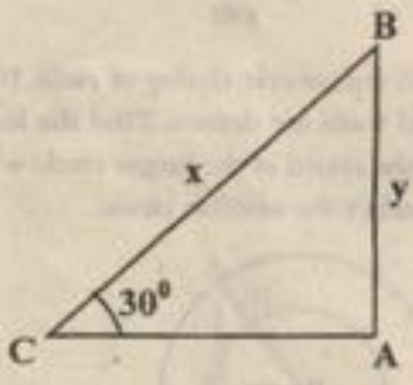
$= 16$cm

Length of the chord AB = 16 cm

16.a) A tree is broken without separating from the stem by the wind. The top touches the ground making an angle 30° at a distance of 12 m from the foot of the tree. Find the height of the tree before breaking.

Sol. Let height of the tree before broken

$= (x + y)$ m



According to the data

Foot of the tree = A

Tree broken at B

Top of the tree touches after broken at C

Given AC = 12 m.

$\angle ACB = 30^\circ$

$\tan 30^\circ = \frac{y}{12}$

$\frac{1}{\sqrt{3}} = \frac{y}{12}$

$y = \frac{12}{\sqrt{3}} = 4\sqrt{3}$m

$\cos 30^\circ = \frac{12}{x}$

$\frac{\sqrt{3}}{2} = \frac{12}{x}$

$x = \frac{12 \times 2}{\sqrt{3}} = 8\sqrt{3}$m

Height of the tree before broken

$= x + y$

$= 8\sqrt{3} + 4\sqrt{3} = 12\sqrt{3}$m

OR

b) How many spherical balls each 7 cm in diameter can be made out of a solid lead cube whose edge measures 66 cm ?

Sol. Given side of cube = a = 66 cm

diameter of sphere = 7 cm

radius of sphere = r = 3.5 cm

Volume of cube = $a^3 = (66)^3$

$= 66 \times 66 \times 66$

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (3.5)^3$

Number of spheres made out from lead cube = $\frac{\text{Volume of cube}}{\text{Volume of sphere}}$

$= \frac{66 \times 66 \times 66}{\frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5}$

$= 1600.16$

= 1600 spheres.

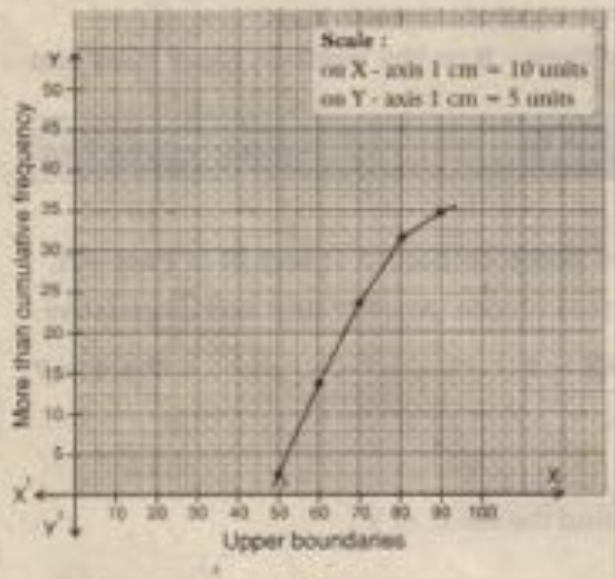
17.a) The literacy rate (in percentage) of 35 cities is given in the following table.

Literacy rate %	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
Number of cities	3	11	10	8	3

Prepare 'more than type' cumulative frequency table and draw ogive curve for this data.

Sol.

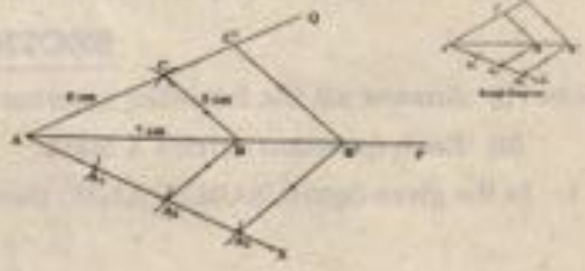
Literacy rate %	Number of cities	Class upper boundaries	More than cumulative frequency
40 – 50	3	50	3
50 – 60	11	60	14
60 – 70	10	70	24
70 – 80	8	80	32
80 – 90	3	90	35



OR

b) Construct a triangle of sides 5 cm, 6 cm and 7 cm. Then construct a triangle similar to it, whose sides are 1½ times the corresponding sides of the first triangle.

Sol.



PART - B

1. A 2. A 3. B 4. D 5. A 6. D 7. B 8. A 9. C

10. A or C

✣ ✣ ✣ ✣ ✣