CLASS - X **TELANGANA**

A GRADE MODEL PAPER 4

MATHEMATICS : PAPER - I

MARCH 2018

Time : 2.45 Hours] **Parts - A and B** [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.
2. Answer the Questions under **Part – A** on a separate answer book.
3. Write the answers to the questions under **Part – B** on the question paper itself and attach it to the answer book of **Part – A**.

Time : 2 Hours] **PART - A** [Marks : 35

Note :

i) Answer **all** the questions from the given **three** sections I, II and III of **Part – A.**

ii) In section – III, every question has internal choice. Answer **any one** alternative.

SECTION – I

(Marks : $7 \times 1 = 7$)

Note : i) Answer **all** the following questions.

ii) Each question carries **1** mark.

1. Find the distance between the points (1, 5) and (5, 8).
2. Expand $\log_{10} 385$.
3. Give one example each for a finite set and an infinite set.
4. Find sum and product of roots of the Quadratic equation.

   $x^2 - 4\sqrt{3}x + 9 = 0$
5. Is the sequence $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$ form an Arithmetic Progression ? Give reason.
6. If $x = a$ and $y = b$ is solution for the pair of equations $x - y = 2$ and $x + y = 4$, then find the values of a and b.
7. Verify the relation between zeroes and coefficients of the Quadratic polynomial

SECTION – II

(Marks : 6 × 2 = 12)

Note : i) Answer **all** the following questions.

ii) Each question carries **2** marks.

8. Complete the following table for the polynomial $y = p(x) = x^3 - 2x + 3$.

x	−1	0	1	2
x^3				
$-2x$				
3				
y				
(x, y)				

9. Show that $\log\frac{162}{343} + 2\log\frac{7}{9} - \log\frac{1}{7} = \log 2$.

10. If the equation $kx^2 - 2kx + 6 = 0$ has equal roots, then find the value of k.

11. Find the 7[th] term from the end of the Arithmetic Progression 7, 10, 13, ..., 184.

12. In the diagram on a Lunar eclipse, if the positions of Sun, Earth and Moon are shown by (– 4, 6), (k, – 2) and (5, – 6) respectively, then find the value of k.

13. Given the linear equation $3x + 4y = 11$, write linear equations in two variables such that their geometrical representations form parallel lines and intersecting lines.

SECTION – III

(Marks : 4 × 4 = 16)

Note : i) Answer **all** the following questions.

ii) In this section, every question has internal choice.

iii) Answer **any one** alternative.

iv) Each question carries **4** marks.

14. Find the points of tri-section of the line segment joining the points (– 2, 1) and (7, 4).

OR

Sum of squares of two consecutive even numbers is 580. Find the numbers by writing a suitable Quadratic equation.

15. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

OR

Show that cube of any positive integer will be in the form of 8m or 8m + 1 or 8m + 3 or 8m + 5 or 8m + 7, where m is a whole number.

16. Find the solution of $x + 2y = 10$ and $2x + 4y = 8$ graphically.

OR

$A = \{x : x \text{ is a perfect square}, x < 50, x \in N)\}$

$B = \{x : x = 8m + 1, \text{ where } m \in W, x < 50, x \in N)\}$

Find $A \cap B$ and display it with Venn diagram.

17. Find the sum of all two digit positive integers which are divisible by 3 but not by 2.

OR

Total number of pencils required are given by $4x^4 + 2x^3 - 2x^2 + 62x - 66$. If each box contains $x^2 + 2x - 3$ pencils, then find the number of boxes to be purchased.

Time : 30 Mts.]	PART - B	[Marks : 5

Note :

(i) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

(ii) Answer **all** the questions.

(iii) Each question carries ½ mark.

(iv) Answers are to be written in question paper only.

(v) Marks will **not** be awarded in any case of overwriting, rewriting or erased answers.

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1. If $-\frac{2}{7}, x, -\frac{7}{2}$ are in Geometric Progression, then the value of x is [ ]

A) 2 B) 1 C) 0 D) 14

2. If $\log_3 729 = x$, then the value of x is [ ]

A) 9 B) 243 C) 81 D) 6

3. Slope of the line passing through the points (4, 6) and (2, – 5) is [ ]

A) $\frac{6}{5}$ B) $\frac{-2}{4}$ C) $\frac{5}{6}$ D) $\frac{11}{2}$

4. The value of k for which the system of equations $kx - y = 2$ and $6x - 2y = 3$ has no solution, is [ ]

A) = 3 B) ≠ 3 C) ≠ 0 D) = 0

5. The number of digits in the fractional part of the decimal form of $\frac{7}{40}$ is [ ]

A) 1 B) 2 C) 3 D) 4

6. If the polynomial $p(x) = x^3 - x^2 + 3x + k$ is divided by $(x - 1)$, the remainder obtained is 3, then the value of k is [ ]

A) 0 B) 1 C) 3 D) – 3

7. If $A \subset B$, $n(A) = 12$ and $n(B) = 20$, then the value of $n(B - A)$ is [ ]

A) 32 B) – 8 C) 8 D) – 32

8. If a number is 132 smaller than its square, then the number is [ ]

A) 11 B) 8 C) 9 D) 12

9. In an Arithmetic Progression, 4th term is 11 and 7th term is 17, then its common difference is [ ]

A) 1 B) 2 C) 3 D) 4

10. In a division, if divisor is x + 1, quotient is x and remainder is 4, then dividend is

A) $x^2 + x$ B) $4(x + 1) + x$ C) $x(x + 1) + 4$ D) $4x + 4$ [ ]

SOLUTIONS

PART - A

SECTION – I

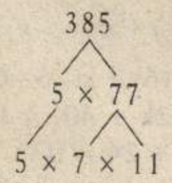
1. **Find the distance between the points (1, 5) and (5, 8).**

Sol. (1, 5) (5, 8)

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(5-1)^2 + (8-5)^2}$$
$$= \sqrt{(4)^2 + (3)^2}$$
$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units.}$$

2. **Expand $\log_{10} 385$.**

Sol. $\log_{10} 385$
$= \log_{10} (5 \times 7 \times 11)$
$= \log_{10} 5 + \log_{10} 7 + \log_{10} 11$



3. **Give one example each for a finite set and an infinite set.**

Sol. Finite set A = {1, 2, 3}
Infinite set B = {1, 2, 3, 4, ...}

4. **Find sum and product of roots of the Quadratic equation.**
$x^2 - 4\sqrt{3}x + 9 = 0$

Sol. Quadratic equation $x^2 - 4\sqrt{3}x + 9 = 0$

$a = 1 ; b = -4\sqrt{3} ; c = 9$

$$\text{Sum of roots} = -\frac{b}{a}$$
$$= \frac{-(-4\sqrt{3})}{1} = 4\sqrt{3}$$
$$\text{Product of roots} = \frac{c}{a} = \frac{9}{1} = 9$$

5. **Is the sequence $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$, ... form an Arithmetic Progression ? Give reason.**

Sol. Given sequence $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$, ...

$$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3}\,(\sqrt{2} - 1)$$
$$a_3 - a_2 = \sqrt{9} - \sqrt{6} = \sqrt{3}\,(\sqrt{3} - \sqrt{2})$$
$$a_2 - a_1 \neq a_3 - a_2$$

So it is not an Arithmetic progression.

6. **If x = a and y = b is solution for the pair of equations x – y = 2 and x + y = 4, then find the values of a and b.**

Sol. $x - y = 2$
$x + y = 4$

a and b are solutions $a - \not{b} = 2$
$a + \not{b} = 4$

on adding $2a = 6$
$a = 3$

$3 + b = 4 \Rightarrow b = 1$

7. **Verify the relation between zeroes and coefficients of the Quadratic polynomial $x^2 - 4$.**

Sol. $p(x) = x^2 - 4$
$= (x + 2)(x - 2)$

So zeroes are 2 and – 2
Sum of zeroes = 2 + (– 2) = 0
Product of zeroes = 2 (– 2) = – 4
$p(x) = x^2 - 4$
$a = 1, \quad b = 0, \quad c = -4$

$$\text{Sum of zeroes} = \frac{-b}{a} = \frac{-0}{1} = 0$$
$$\text{Product of zeroes} = \frac{c}{a} = \frac{-4}{1} = -4$$

SECTION - II

8. Complete the following table for the polynomial $y = p(x) = x^3 - 2x + 3$.

x	– 1	0	1	2
x^3				
– 2x				
3				
y				
(x, y)				

Sol.

x	– 1	0	1	2
x^3	– 1	0	1	8
– 2x	2	0	– 2	– 4
3	3	3	3	3
y	4	3	2	7
(x, y)	(– 1, 4)	(0, 3)	(1, 2)	(2, 7)

9. Show that

$$\log\frac{162}{343} + 2\log\frac{7}{9} - \log\frac{1}{7} = \log 2.$$

Sol. LHS $= \log\frac{162}{343} + 2\log\frac{7}{9} - \log\frac{1}{7}$

$= \log\left(\frac{3^4 \times 2}{7^3}\right) + 2\log\left(\frac{7}{3^2}\right) - \log\left(\frac{1}{7}\right)$

$= \log 3^4 + \log 2 - \log 7^3$

$+ 2[\log 7 - \log 3^2] - [\log 1 - \log 7]$

$= 4\log3 + \log 2 - 3\log7 + 2\log7$

$- 4\log3 - \log1 + \log7$

$= \log 2$ ($\because \log 1 = 0$)

= RHS

Hence proved

10. If the equation $kx^2 - 2kx + 6 = 0$ has equal roots, then find the value of k.

Sol. Given equation $kx^2 - 2kx + 6 = 0$

It has equal roots

So Discriminent (D) $b^2 - 4ac = 0$

$a = k$; $b = -2k$; $c = 6$

$(-2k)^2 - 4(k)(6) = 0$

$4k^2 = 4 \times 6k$

$k = 6$

11. Find the 7th term from the end of the Arithmetic Progression 7, 10, 13, ..., 184.

Sol. Given Arithmetic progression

7, 10, 13, ..., 184.

Writing it in the reverse

184, 181, ..., 13, 10, 7

$a = 184$; $d = 181 - 184 = -3$.

$a_n = a + (n - 1)d$

7^{th} term so $n = 7$

$a_7 = 184 + (7 - 1)(-3)$

$= 184 - 6(3) = 184 - 18 = 166$

12. In the diagram on a Lunar eclipse, if the positions of Sun, Earth and Moon are shown by (– 4, 6), (k, – 2) and (5, – 6) respectively, then find the value of k.

Sol. Points (– 4, 6) (k, – 2) (5, – 6)

$x_1\ y_1 \quad x_2\ y_2 \quad x_3\ y_3$

Since it is a lunar eclipse, the position of sun, Earth and Moon must be collinear.

Since they are collinear, area of triangle must be zero.

$$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$$

$\Rightarrow |-4(-2 + 6) + k(-6 - 6) + 5(6 + 2)| = 0$

$\Rightarrow |-16 - 12k + 40| = 0$

$\Rightarrow -12k + 24 = 0$

$\Rightarrow 12k = 24 \Rightarrow k = \frac{24}{12} = 2$

13. Given the linear equation $3x + 4y = 11$, write linear equations in two variables such that their geometrical representations form parallel lines and intersecting lines.

Sol. $3x + 4y - 11 = 0$

$a_1 = 3,\ b_1 = 4,\ c_1 = -11$

Condition for parallel lines

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Eg : $6x + 8y - 25 = 0 \left[\because \frac{3}{6} = \frac{4}{8} \neq \frac{-11}{-25}\right]$

Condition for intersecting lines

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Eg : $4x + 3y + 7 = 0 \left[\because \frac{3}{4} \neq \frac{4}{3}\right]$

SECTION - III

14. Find the points of tri-section of the line segment joining the points (– 2, 1) and (7, 4).

Sol. Given points (– 2, 1) (7, 4)

$x_1 = -2, x_2 = 7, y_1 = 1, y_2 = 4$

Points of trisection means the points which divide it in the ratio 1 : 2 and 2 : 1

Section formula

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

Point which divides it in the ratio 1 : 2 is

$$= \left(\frac{1(7) + 2(-2)}{1+2}, \frac{1(4) + 2(1)}{1+2}\right)$$

$$= \left(\frac{7-4}{3}, \frac{4+2}{3}\right) = \left(\frac{3}{3}, \frac{6}{3}\right) = (1, 2)$$

Point which divides it in the ratio 2 : 1 is

$$= \left(\frac{2(7) + 1(-2)}{2+1}, \frac{2(4) + 1(1)}{2+1}\right)$$

$$= \left(\frac{14-2}{3}, \frac{8+1}{3}\right) = \left(\frac{12}{3}, \frac{9}{3}\right) = (4, 3)$$

∴ Points of trisection are (1, 2) and (4, 3).

OR

Sum of squares of two consecutive even numbers is 580. Find the numbers by writing a suitable Quadratic equation.

Sol. Let the two consecutive even numbers be x and (x + 2)

$(x)^2 + (x + 2)^2 = 580$

$x^2 + x^2 + 4x + 4 = 580$

$2x^2 + 4x - 576 = 0$

$x^2 + 2x - 288 = 0$. Quadratic equation.

$x^2 + 18x - 16x - 288 = 0$

$x(x + 18) - 16(x + 18) = 0$

$(x + 18)(x - 16) = 0$

$x = -18$ or $x = +16$

So the required numbers are

– 18, – 16 (or) 16, 18

15. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Sol. Suppose $\sqrt{3} + \sqrt{5}$ is not an irrational number.

Then $\sqrt{3} + \sqrt{5}$ must be a rational number.

$\sqrt{3} + \sqrt{5} = \frac{p}{q}$, $q \neq 0$ and $p, q \in Z$

Squaring on both sides

$$3 + 5 + 2\sqrt{15} = \frac{p^2}{q^2}$$

$$2\sqrt{15} = \frac{p^2}{q^2} - 8 = \frac{p^2 - 8q^2}{q^2}$$

$$\sqrt{15} = \frac{p^2 - 8q^2}{2q^2}$$

but $\sqrt{15}$ is an irrational number.

$\frac{p^2 - 8q^2}{2q^2}$ is a rational number

$(p^2 - 8q^2, 2q^2 \in Z, 2q^2 \neq 0)$

but an irrational number can't be equal to a rational number, so our supposition that $\sqrt{3} + \sqrt{5}$ is not an irrational number is false.

∴ $\sqrt{3} + \sqrt{5}$ is an irrational number.

OR

Show that cube of any positive integer will be in the form of 8m or 8m + 1 or 8m + 3 or 8m + 5 or 8m + 7, where m is a whole number.

Sol. $a = bq + r, 0 \leq r < b$

$a = 8k + t$ for $t = 0, 1, 2, 3, 4, 5, 6, 7$.

$a^3 = (8k + t)^3$

$= (8k)^3 + 3(8k)(t)(8k + t) + (t)^3$

$= 8[64k^3 + 3kt(8k + t)] + t^3$

$= 8n + t^3$

If $t = 0, 2, 4, 6$ then $t^3 = 8p$

$a^3 = (8k + t)^3 = 8n + 8p = 8(n + p) = 8m$

If $t = 1$ then $a^3 = 8n + 1 = 8m + 1$

If $t = 3$ then $a^3 = 8n + 27$

$= 8(n + 3) + 3 = 8m + 3$

If $t = 5$ then $a^3 = 8n + 125$

$= 8(n + 15) + 5 = 8m + 5$

If $t = 7$ then $a^3 = 8n + 343$

$= 8(n + 42) + 7 = 8m + 7$

∴ The cube of any positive integer will be of the form 8m or 8m +1 or 8m +3 or 8m + 5 or 8m +7.

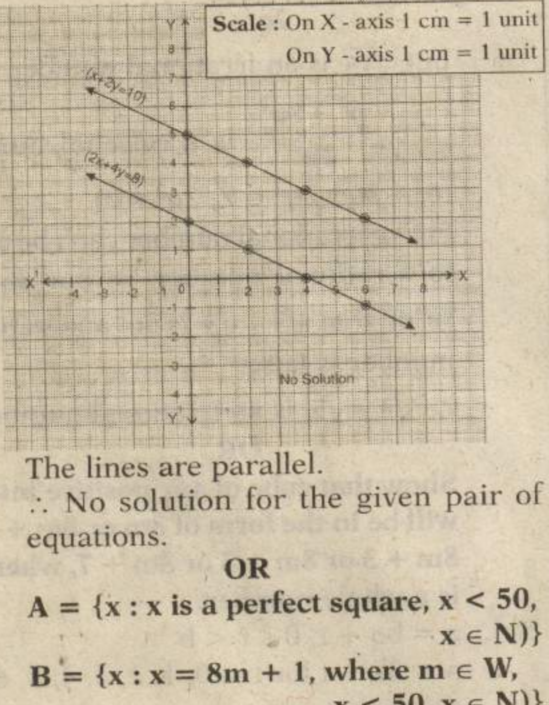
16. Find the solution of $x + 2y = 10$ and $2x + 4y = 8$ graphically.

Sol. $x + 2y = 10$ .... (1)

x	0	2	4	6
y	5	4	3	2
(x, y)	(0, 5)	(2, 4)	(4, 3)	(6, 2)

$2x + 4y = 8$ .... (2)

x	0	2	4	6
y	2	1	0	−1
(x, y)	(0, 2)	(2, 1)	(4, 0)	(6, −1)



The lines are parallel.

$\therefore$ No solution for the given pair of equations.

OR

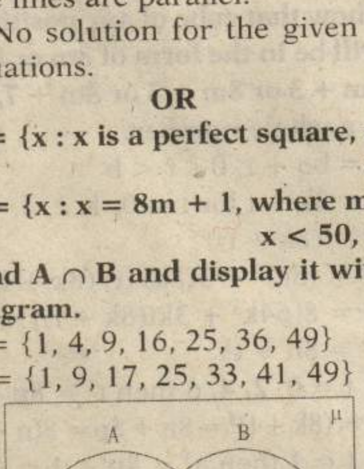
A = {x : x is a perfect square, x < 50, x ∈ N)}

B = {x : x = 8m + 1, where m ∈ W, x < 50, x ∈ N)}

Find A ∩ B and display it with Venn diagram.

Sol. A = {1, 4, 9, 16, 25, 36, 49}

B = {1, 9, 17, 25, 33, 41, 49}



A ∩ B = {1, 9, 25, 49}

17. Find the sum of all two digit positive integers which are divisible by 3 but not by 2.

Sol. Two digit numbers which are divisible by 3 but not by 2 are 15, 21, 27, ..., 99

$21 - 15 = 6$; $27 - 21 = 6$

$\therefore$ These terms are in A.P.

$a = 15, d = 6, a_n = 99$

$a_n = a + (n - 1)d$

$99 = 15 + (n - 1)6$

$n - 1 = \frac{84}{6} = 14$

$n = 15$

$S_n = \frac{n}{2}[a + l]$

$= \frac{15}{2}[15 + 99] = \frac{15}{2} \times 114 = 855$

OR

Total number of pencils required are given by $4x^4 + 2x^3 - 2x^2 + 62x - 66$. If each box contains $x^2 + 2x - 3$ pencils, then find the number of boxes to be purchased.

Sol. Total number of pencils

$= 4x^4 + 2x^3 - 2x^2 + 62x - 66$

Number of pencils in each box

$= x^2 + 2x - 3.$

Number of boxes required $= (4x^4 + 2x^3 - 2x^2 + 62x - 66) \div (x^2 + 2x - 3)$

$x^2 + 2x - 3 \,)\, 4x^4 + 2x^3 - 2x^2 + 62x - 66 \,(\, 4x^2 - 6x + 22$

$4x^4 + 8x^3 - 12x^2$

$(-) \quad (-) \quad (+)$

$-6x^3 + 10x^2 + 62x$

$-6x^3 - 12x^2 + 18x$

$(+) \quad (+) \quad (-)$

$22x^2 + 44x - 66$

$22x^2 + 44x - 66$

$(-) \quad (-) \quad (+)$

0

$\therefore$ Number of boxes required

$= 4x^2 - 6x + 22$

PART - B

1) B 2) D 3) D 4) A 5) C 6) A 7) C 8) D 9) B 10) C

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