CLASS - X

TELANGANA

A GRADE MODEL PAPER 6

MATHEMATICS : PAPER - I

MARCH 2017

Time : 2 hours 45 min.] **Parts - A and B** [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. *In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.*
2. *Answer the Questions under '**Part – A**' on a separate answer book.*
3. *Write the answers to the Questions under '**Part – B**' on the Question paper itself and attach it to the answer book of '**Part – A**'.*

Time : 2 Hours] **PART - A** **[Marks : 35**

Note :

1. *Answer **all** the questions from the given **three** sections – I, II and III of **Part – A.***
2. *In section – III, every question has internal choice. Answer **anyone** alternative.*

SECTION – I

(Marks : 7 × 1 = 7)

Note : (i) Answer **all** the following questions.

(ii) Each question carries **1** mark.

1. Write the nature of roots of the quadratic equation $2x^2 - 5x + 6 = 0$.
2. Find the value of $\log_{\sqrt{2}} 256$.
3. In a GP, $t_n = (-1)^n . 2017$. Find the common ratio.
4. Srikar says that the order of the polynomial $(x^2 - 5)(x^3 + 1)$ is 6. Do you agree with him? How ?
5. A (0, 3), B (k, 0) and AB = 5. Find the positive value of k.
6. Show that the pair of linear equations $7x + y = 10$ and $x + 7y = 10$ are consistent.
7. Represent $A \cap B$ through Venn diagram, where A = {1, 4, 6, 9, 10} and B = {perfect squares less than 25}.

SECTION – II

(Marks : 6 × 2 = 12)

Note : *(i) Answer **all** the following questions.*

*(ii) Each question carries **2** marks.*

8. Write any two three digit numbers. Find their L.C.M. and G.C.D. by prime factorization method.
9. Find the sum of first 10 terms of an A.P.

   3, 15, 27, 39, ...
10. Which of $\sqrt{2}$ and 2 is a zero of the polynomial $p(x) = x^3 - 2x$? Why ?
11. The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number.
12. Two vertices of a triangle are (3, 2), (–2, 1) and its centroid is $\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the third vertex of the triangle.
13. Find the angle made by the line joining (5, 3) and (–1, –3) with the positive direction of X – axis.

SECTION – III

(Marks : 4 × 4 = 16)

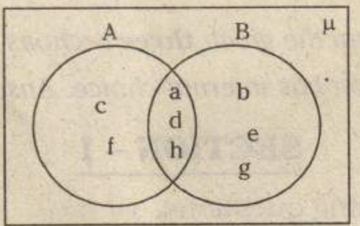
Note : *(i) Answer **all** the following questions.*

(ii) In this section, every question has internal choice.

*(iii) Answer **anyone** alternative.*

*(iv) Each question carries **4** marks.*

14. From the following Venn diagram, write the elements of the sets of A and B. And verify $n(A \cup B) + n(A \cap B) = n(A) + n(B)$.



OR

Use Euclid's division lemma to show that the square of any positive integer is of the form 5n or 5n + 1 or 5n + 4, where n is a whole number.

15. Find the sum of all three digit natural numbers, which are divisible by 3 and not divisible by 6.

OR

Divide $3x^4 - 5x^3 + 4x^2 + 3x - 5$ by $x^2 - 3$ and verify the division algorithm.

16. The perimeter of a right-angled triangle is 60 cm. and its hypotenuse is 25 cm. Then find the remaining two sides.

OR

The points C and D are on the line segment joining A (– 4, 7) and B (5, 13) such that AC = CD = DB. Then find coordinates of points C and D.

17. Draw the graph for the polynomial $p(x) = x^2 - 5x + 6$ and find the zeroes from the graph.

OR

Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$ and find the solution from the graph.

Time : 30 Minutes]	PART - B	[Marks : 5

Instructions :

(i) *Write the answers to the questions in this **Part-B** on the Question paper itself and attach it to the answer book of **Part-A**.*

(ii) *Answer **all** the questions.*

(iii) *Each question carries ½ mark.*

(iv) *Answers are to be written in question paper only.*

(v) *Marks will **not** be awarded in any case of over - writing, rewriting or erased answers.*

I. Write the **CAPITAL LETTERS (A,B,C,D)** showing the correct answer for the following questions in the brackets provided against them. ***(Marks : 10 × ½ = 5)***

1. If P_1 and P_2 are two odd prime numbers, such that $P_1 > P_2$, then $P_1^2 - P_2^2$ is .... [ ]

(A) an even number. (B) an odd number.

(C) a prime number. (D) an odd prime number.

2. Sum of the distances from A (3, 4) to X–axis and from B (5, 7) to Y–axis is ..... [ ]

(A) 8 (B) 10 (C) 11 (D) 9

3. $S = \left\{3, \pi, \sqrt{2}, -5, 3 + \sqrt{7}, \frac{2}{7}\right\}$. Which of the following is a sub-set of 'S' containing all irrational numbers ? [ ]

(A) $\left\{3, \pi, \frac{2}{7}, -5, 3 + \sqrt{7}\right\}$ (B) $\{3 + \sqrt{7}, \sqrt{2}, \pi\}$

(C) $\{3, \pi, \sqrt{2}\}$ (D) $\left\{3, -5, \frac{2}{7}\right\}$

4. Which of the following is inconsistent equation to $2x + 3y - 5 = 0$? [ ]

(A) $4x - 6y - 11 = 0$ (B) $2x + y = 5$

(C) $x + 3y = 5$ (D) $4x + 6y - 11 = 0$

5. Sum of zeroes of a polynomial $x^3 - 2x^2 + 3x - 4$ is .... [ ]

(A) -2 (B) 2 (C) 1 (D) 4

6. (x, y), (2, 0), (3, 2) and (1, 2) are vertices of a parallelogram, then (x, y) = ... [ ]

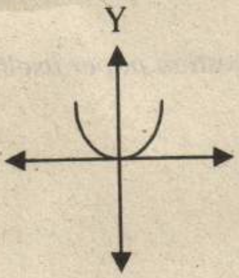
(A) (0, 0) (B) (4, 8) (C) (1, 0) (D) (5, 0)

7. If $x^2 - px + q = 0$ ($p, q \in R$ and $p \neq 0, q \neq 0$) has distinct real roots, then .... [ ]

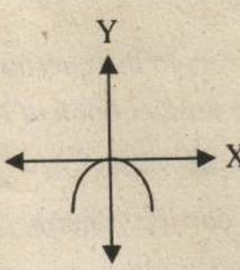
(A) $p^2 < 4q$ (B) $p^2 > 4q$ (C) $p^2 = 4q$ (D) $p^2 + 4q = 0$

8. **The graph represented by y = x is ....** [ ]

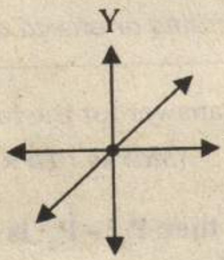
(A)



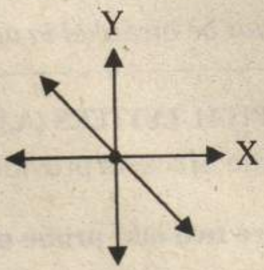
(B)



(C)



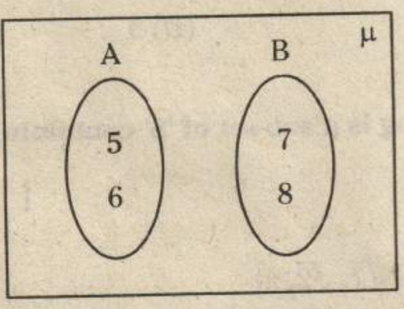
(D)



9. **The distance between two points A (a cos θ, 0), B (0, a sin θ) is ....** [ ]

(A) a (B) a^2 (C) $\sqrt{a}$ (D) 0

10.



From the Venn diagram, A ∪ B = ... [ ]

(A) {5, 6} (B) {5, 6, 7, 8} (C) ∅ (D) {7, 8}

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SOLUTIONS

PART - A

SECTION – I

1. Write the nature of roots of the quadratic equation $2x^2 - 5x + 6 = 0$.

Sol. Discriminent (D) = $b^2 - 4ac$

$= (-5)^2 - 4(2)(6)$

$= 25 - 48$

$= -23$

D < 0, So roots are not Real.

2. Find the value of $\log_{\sqrt{2}} 256$.

Sol. $256 = 2^8$, $\sqrt{2} = 2^{\frac{1}{2}}$

$\log_{\sqrt{2}} 256 = \log_{2^{\frac{1}{2}}} 2^8$

$= \dfrac{8}{\frac{1}{2}} \log_2 2$

$\left(\because \log_{b^n} a^m = \dfrac{m}{n} \log_b a\right)$

$= 8 \times 2 \times 1$ ($\because \log_2 2 = 1$) $= 16$

3. In a GP, $t_n = (-1)^n$. 2017. Find the common ratio.

Sol. $t_n = (-1)^n 2017$

$t_1 = (-1)^1 2017 = -2017$

$t_2 = (-1)^2\ 2017 = 2017$

$r = \dfrac{t_2}{t_1} = \dfrac{2017}{-2017} = -1$

4. Srikar says that the order of the polynomial $(x^2 - 5)(x^3 + 1)$ is 6. Do you agree with him? How ?

Sol. $(x^2 - 5)(x^3 + 1) = x^5 + x^2 - 5x^3 - 5$

As its degrée is 5, I do not agree with Srikar.

5. A (0, 3), B (k, 0) and AB = 5. Find the positive value of k.

Sol. Distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$5 = \sqrt{(k-0)^2 + (0-3)^2}$

$25 = k^2 + 9$

$k^2 = 16$

$k = \pm 4$

$\therefore$ Positive value of k is 4.

6. Show that the pair of linear equations $7x + y = 10$ and $x + 7y = 10$ are consistent.

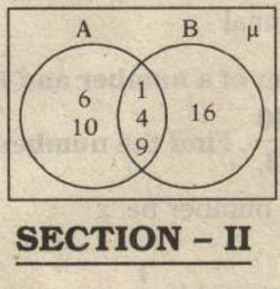
Sol. If $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$, then the equations are consistent

Since $\dfrac{7}{1} \neq \dfrac{1}{7}$, the given equations are consistent

7. Represent $A \cap B$ through Venn diagram, where A = {1, 4, 6, 9, 10} and B = {perfect squares less than 25}.

Sol. A = {1, 4, 6, 9, 10}

B = {1, 4, 9, 16}



SECTION – II

8. Write any two three digit numbers. Find their L.C.M. and G.C.D. by prime factorization method.

Sol. Let the two three digit numbers be 100 and 200

$100 = 2^2 \times 5^2$

$200 = 2^3 \times 5^2$

L.C.M. $= 2^3 \times 5^2 = 200$

G.C.D $= 2^2 \times 5^2 = 100$

9. **Find the sum of first 10 terms of an A.P.**

3, 15, 27, 39, ...

Sol. $a = 3$

$d = 15 - 3 = 12$

$n = 10$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2(3) + (10-1)12]$$

$= 5[6 + 108]$

$= 5 \times 114$

$= 570$

10. **Which of $\sqrt{2}$ and 2 is a zero of the polynomial $p(x) = x^3 - 2x$? Why ?**

Sol. $p(x) = x^3 - 2x$

$p(\sqrt{2}) = (\sqrt{2})^3 - 2(\sqrt{2})$

$= 2\sqrt{2} - 2\sqrt{2} = 0$

$p(2) = (2)^3 - 2(2)$

$= 8 - 4 = 4$

Since $p(\sqrt{2}) = 0$, $\sqrt{2}$ is a zero of the polynomial

11. **The sum of a number and its reciprocal is $\frac{10}{3}$. Find the number.**

Sol. Let the number be 'x'

Its reciprocal is $\frac{1}{x}$

$x + \frac{1}{x} = \frac{10}{3}$, the quadratic equation obtained from this is $3x^2 - 10x + 3 = 0$

After solving, we get $x = 3$ or $\frac{1}{3}$

$\therefore$ Required number is 3 or $\frac{1}{3}$.

12. **Two vertices of a triangle are (3, 2), (–2, 1) and its centroid is $\left(\frac{5}{3}, -\frac{1}{3}\right)$. Find the third vertex of the triangle.**

Sol. Let the third vertex be (x, y)

Centroid (G)

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

$$\left(\frac{5}{3}, -\frac{1}{3}\right) = \left(\frac{3 + (-2) + x}{3}, \frac{2 + 1 + y}{3}\right)$$

$\frac{5}{3} = \frac{1 + x}{3}$ | $\frac{3 + y}{3} = -\frac{1}{3}$

$x = 4$ | $y = -4$

So third vertex is (4, – 4)

13. **Find the angle made by the line joining (5, 3) and (–1, –3) with the positive direction of X – axis.**

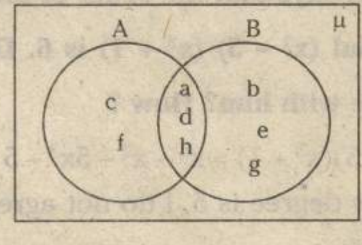
Sol. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{-1 - 5} = \frac{-6}{-6} = 1$

$\tan\theta = 1 = \tan 45^\circ$

Required angle $(\theta) = 45^\circ$

SECTION – III

14. **From the following Venn diagram, write the elements of the sets of A and B. And verify $n(A \cup B) + n(A \cap B) = n(A) + n(B)$.**



Sol. A = {a, c, d, f, h}

$B = \{a, b, d, e, g, h\}$

$A \cup B = \{a, b, c, d, e, f, g, h\}$

$A \cap B = \{a, d, h\}$

$n(A) = 5, n(B) = 6, n(A \cup B) = 8,$

$n(A \cap B) = 3$

$L.H.S = n(A \cup B) + n(A \cap B) = 8 + 3 = 11$

$R.H.S = n(A) + n(B) = 5 + 6 = 11$

$\therefore n(A \cup B) + n(A \cap B) = n(A) + n(B)$

OR

Use Euclid's division lemma to show that the square of any positive integer is of the form 5n or 5n + 1 or 5n + 4, where n is a whole number.

Sol. $a = bq + r, 0 \le r < b$

$b = 5$ so $r = 0, 1, 2, 3, 4$

Then 'a' can be of the forms

$5q + 0, 5q + 1, 5q + 2, 5q + 3, 5q + 4$

Case (i) When $a = 5q$

$a^2 = (5q)^2 = 5(5q^2) = 5n$ where

$n = 5q^2 \in W$

Case (ii) When $a = 5q + 1$

$a^2 = (5q + 1)^2$

$= 25q^2 + 10q + 1$

$= 5(5q^2 + 2q) + 1$

$= 5n + 1$ where $n = 5q^2 + 2q \in W$

Similarly

Case (iii) $a^2 = (5q + 2)^2 = 5n + 4$

Case (iv) $a^2 = (5q + 3)^2 = 5n + 4$

Case (v) $a^2 = (5q + 4)^2 = 5n + 1$

So the square of any positive integer is of the form 5n or 5n + 1 or 5n + 4 where $n \in W$

15. Find the sum of all three digit natural numbers, which are divisible by 3 and not divisible by 6.

Sol. Three digit numbers which are divisible by 3 and not divisible by 6 are

105, 111, 117, ... 999

$a = 105$ $d = 111 - 105 = 6$

$l = 999$

$a_n = a + (n - 1)d$

$999 = 105 + (n - 1)6$

$$n - 1 = \frac{999 - 105}{6} = 149$$

$\therefore n = 150$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{150} = \frac{150}{2}[105 + 999]$$

$= 75 \times 1104 = 82,800$

OR

Divide $3x^4 - 5x^3 + 4x^2 + 3x - 5$ by $x^2 - 3$ and verify the division algorithm.

Sol. $x^2 - 3)\ 3x^4 - 5x^3 + 4x^2 + 3x - 5\ (3x^2 - 5x + 13$

$3x^4 \quad - 9x^2$

$(-) \quad (+)$

$- 5x^3 + 13x^2 + 3x$

$- 5x^3 \quad + 15x$

$(+) \quad (-)$

$13x^2 - 12x - 5$

$13x^2 \quad - 39$

$(-) \quad (+)$

$- 12x + 34$

Verification :

(Divisor × Quotient) + remainder = Dividend.

$(x^2 - 3)(3x^2 - 5x + 13) + (-12x + 34)$

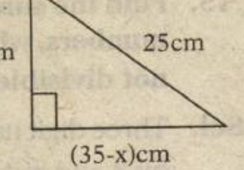
$= 3x^4 - 5x^3 + 4x^2 + 3x - 5.$

16. The perimeter of a right-angled triangle is 60 cm. and its hypotenuse is 25 cm. Then find the remaining two sides.

Sol. Let one side be 'x' cm

Remaining side = 60–(25 + x)

= (35 – x) cm



Using Pythagoras theorem,

$x^2 + (35 - x)^2 = (25)^2$

After simplifications, we get

$x^2 - 35x + 300 = 0$

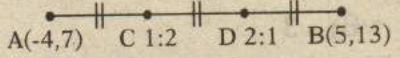
After solving, we get x = 15 or 20

∴ The remaining two sides of the triangle measures 15 cm and 20 cm.

OR

The points C and D are on the line segment joining A (– 4, 7) and B (5, 13) such that AC = CD = DB. Then find coordinates of points C and D.

Sol.



Section Formula

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

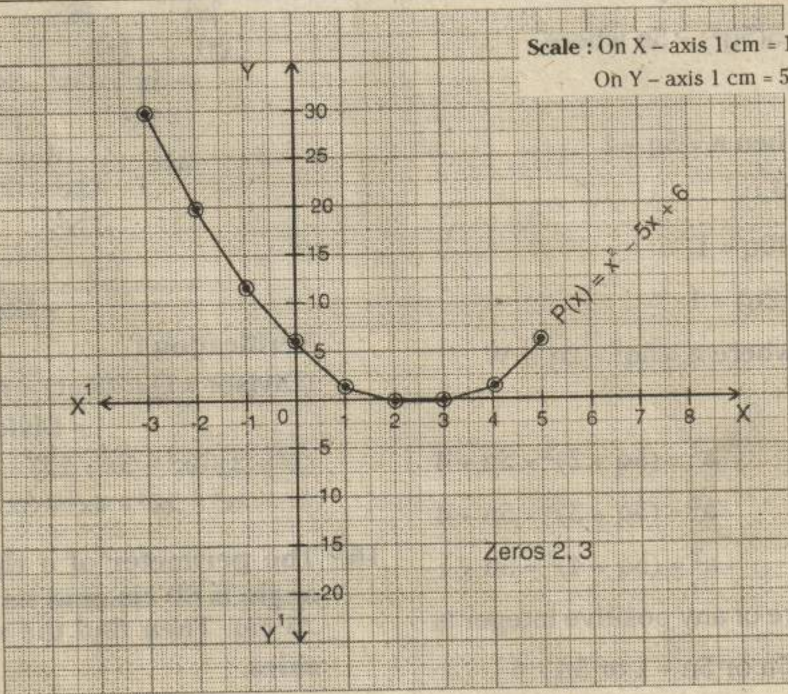
$$C = \left(\frac{1(5) + 2(-4)}{1+2}, \frac{1(13) + 2(7)}{1+2}\right) = (-1, 9)$$

$$D = \left(\frac{2(5) + 1(-4)}{2+1}, \frac{2(13) + 1(7)}{2+1}\right) = (2, 11)$$

17. Draw the graph for the polynomial $p(x) = x^2 - 5x + 6$ and find the zeroes from the graph.

Sol. $p(x) = x^2 - 5x + 6$

x	–3	–2	–1	0	1	2	3	4	5
x^2	9	4	1	0	1	4	9	16	25
–5x	15	10	5	0	–5	–10	–15	–20	–25
6	6	6	6	6	6	6	6	6	6
y	30	20	12	6	2	0	0	2	6
(x, y)	(–3,30)	(–2,20)	(–1,12)	(0,6)	(1,2)	(2,0)	(3,0)	(4,2)	(5,6)



Zeroes of polynomial are 2 and 3.

OR

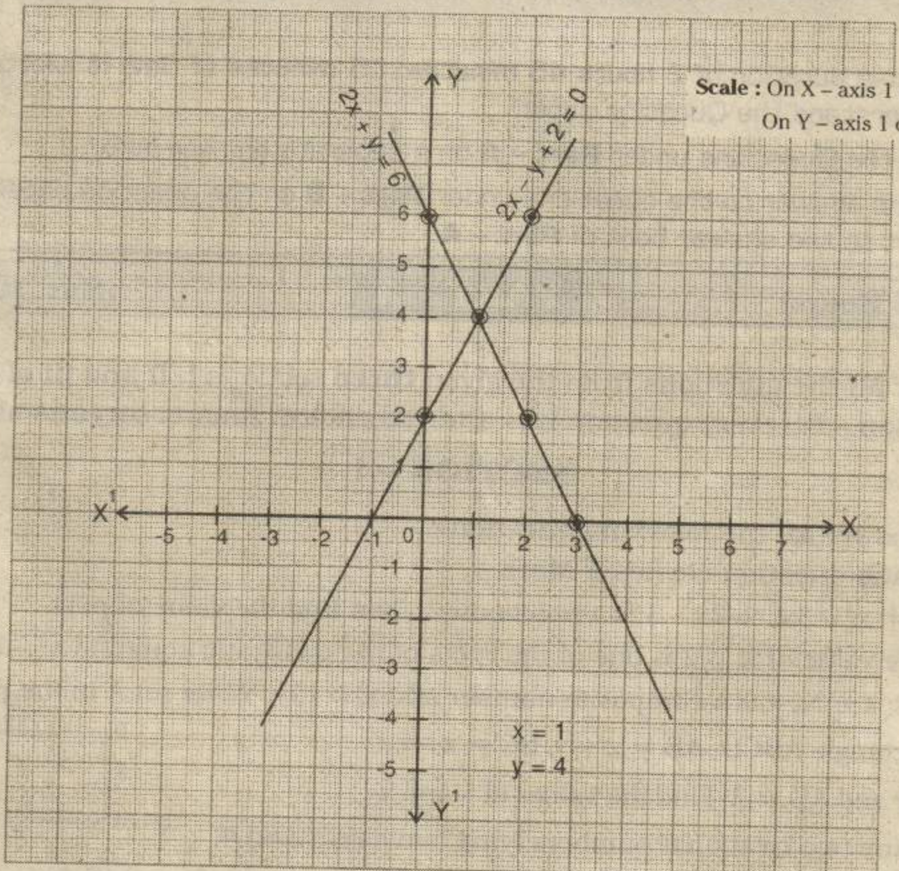
Draw the graph of $2x + y = 6$ and $2x - y + 2 = 0$ and find the solution from the graph.

Sol. $2x + y = 6 \rightarrow (1)$

x	0	1	2	3
y	6	4	2	0
(x, y)	(0, 6)	(1, 4)	(2, 2)	(3, 0)

$2x - y + 2 = 0 \rightarrow (2)$

x	0	1	2
y	2	4	6
(x, y)	(0, 2)	(1, 4)	(2, 6)



Intersecting point of equations (1) and (2) is (1, 4)

So $x = 1$; $y = 4$

PART - B

1) A 2) D 3) B 4) D 5) B 6) A 7) B 8) C 9) A 10) B