CLASS - X

TELANGANA

MODEL PAPER 1

MATHEMATICS : PAPER - I

JUNE 2019

Time : 2.45 Hours] **Parts - A and B** [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.
2. Answer **all** the Questions under **Part – A** on a separate answer book.
3. Write the answers to the questions under **Part – B** on the Question paper itself and attach it to the answer book of **Part – A**.

Time : 2 Hours] **PART - A** [Marks : 35

Note :

(i) Answer **all** the questions from the given **three** sections **I, II** and **III** of **Part – A.**

(ii) In section III, every question has internal choice. Answer **any one** alternative.

SECTION – I

(Marks : 7 × 1 = 7)

Note : (i) Answer **all** the following questions.

(ii) Each question carries **1** mark.

1. Ramu says, "If $\log_{10} x = 0$, value of $x = 0$". Do you agree with him ? Give reason.
2. Determine 'x' so that 2 is the slope of the line passing through A (–2, 4) and B(x, –2).
3. –3, 0 and 2 are the zeroes of the polynomial $p(x) = x^3 + (a-1)x^2 + bx + c$, find a and c.
4. Find the discriminant of the quadratic equation $3x^2 - 5x + 2 = 0$ and hence write the nature of its roots.
5. Find the 11^{th} term of the A.P. : 15, 12, 9, ...
6. If A = {1, 2, 3}, B = {3, 4, 5}, then find A – B and B – A.
7. Write any two linear polynomials having one term and three terms.

SECTION – II

(Marks : 6 × 2 = 12)

Note : (i) Answer **all** the following questions.

(ii) Each question carries **2** marks.

8. If A = {x : x is a factor of 12} and
   B = {x : x is a factor of 6},
   then find $A \cup B$ and $A \cap B$.

9. Find the roots of quadratic equation $x^2 + 4x + 3 = 0$ by "Completing Square method".

10. For what value of 'm' in the following,
$mx + 4y = 10$ and $9x + 12y = 30$ system of equations will have no solution ? Why ?

11. Which term of the G.P. : $\sqrt{2}, 2, 2\sqrt{2}, 4, ...$ is 32 ?

12. If $x^2 + y^2 = 10xy$, prove that $2 \log (x + y) = \log x + \log y + 2 \log 2 + \log 3$.

13. Shashanka said that $(x + 1)^2 = 2(x - 3)$ is a quadratic equation. Do you agree ?

SECTION – III

(Marks : 4 × 4 = 16)

Note : i) Answer **all** the following questions.

ii) In this section, every question has internal choice.

iii) Answer **any one** alternative.

iv) Each question carries **4** marks.

14. Use division algorithm to show that the square of any positive integer is of the form $5m$ or $5m + 1$ or $5m + 4$, where 'm' is a whole number.

OR

Show that $\sqrt{5} - \sqrt{3}$ is an irrational number.

15. Draw the graph for the polynomial $p(x) = x^2 + 3x - 4$ and hence find the zeroes from the graph.

OR

Draw the graph of $x + y = 11$ and $x - y = 5$. Find the solution of the pair of linear equations.

16. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

Find the sum of all the integers between 1 to 50 which are not divisible by 3.

17. Find the area of a rhombus ABCD, whose vertices taken in order, are
A (–1, 1), B(1, –2), C(3, 1) and D(1, 4).

OR

If $A = \{x : x \text{ is a prime less than } 20\}$ and

$B = \{x : x \text{ is a whole number less than } 10\}$,

then verify $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Time : 30 Mts.]	**PART - B**	[Marks : 5

Note :

(i) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

(ii) Answer **all** the questions.

(iii) Each question carries ½ mark.

(iv) Answers are to be written in question paper only.

(v) Marks will **not** be awarded in any case of overwriting, rewriting or erased answers.

1. If a, b, c are in A.P., then b = [ ]

A) $\frac{a+c}{2}$ B) $a + c$ C) $\sqrt{ac}$ D) ac

2. If the number of subsets of a given set is 32, then the number of elements in the set will be [ ]

A) 2 B) 4 C) 5 D) 3

3. The distance of (3, 4) from origin is [ ]

A) 3 B) 4 C) 5 D) 7

4. The sum of the roots of $6x^2 = 1$ is [ ]

A) 0 B) $\frac{1}{6}$ C) $-\frac{1}{6}$ D) 6

5. If the polynomial $p(x) = x^4 - 2x^3 + x^2 - 1$ is divided by $(x + 1)$, then the degree of quotient polynomial. [ ]

A) 1 B) 3 C) 4 D) 2

6. The sum of a number and reciprocal is $\frac{17}{4}$, then the number is [ ]

A) 3 B) 4 C) 5 D) 17

7. If $\log_{10}2 = 0.3010$, then $\log_{10}32$ is [ ]

A) 5.3010 B) 2.3010 C) 1.5050 D) 0.3010

8. The point (–2, –2) is in the .... quadrant. [ ]

A) Q_1 B) Q_2 C) Q_3 D) Q_4

9. The sum of the first 20 even natural numbers is [ ]

A) 5050 B) 55 C) 505 D) 420

10. The roots of a quadratic equation $ax^2 - bx + c = 0, a \neq 0$ are [ ]

A) $\frac{-b+\sqrt{b^2-4ac}}{2a}; \frac{-b+\sqrt{b^2+4ac}}{2a}$ B) $\frac{-b+\sqrt{b^2-4ac}}{2a}; \frac{-b-\sqrt{b^2+4ac}}{2a}$

C) $\frac{b+\sqrt{b^2-4ac}}{2a}; \frac{b-\sqrt{b^2-4ac}}{2a}$ D) $\frac{-b+\sqrt{b^2-4ac}}{2a}; \frac{-b-\sqrt{b^2-4ac}}{2a}$

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SOLUTIONS

PART - A

SECTION – I

1. Ramu says, "If $\log_{10} x = 0$, value of $x = 0$". Do you agree with him ? Give reason.

Sol. $\log_{10}x = 0$ $\quad [\because \log_a N = x \Rightarrow a^x = N]$

$10^0 = x$

$x = 1$ & $x \neq 0$

$\therefore$ I don't agree with Ramu.

2. Determine 'x' so that 2 is the slope of the line passing through A (–2, 4) and B(x, –2).

Sol. Given slope of the line passing through A(–2, 4) and B (x, –2) is 2.

Here $x_1 = -2; y_1 = 4; x_2 = x; y_2 = -2$

Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = 2$.

$$\frac{-2-4}{x-(-2)} = 2 \Rightarrow \frac{-6}{x+2} = 2$$

$$\Rightarrow -6 = 2(x + 2)$$

$$\Rightarrow -6 = 2x + 4$$

$$2x = -10 \Rightarrow x = \frac{-10}{2} = -5$$

$\therefore x = -5$

3. –3, 0 and 2 are the zeroes of the polynomial $p(x) = x^3 + (a-1)x^2 + bx + c$, find a and c.

Sol. Given solution compare with $ax^3 + bx^2 + cx + d$

Given equation $p(x) = x^3 + (a-1)x^2 + bx + c$.

Given roots –3, 0, 2

Sum of the roots $\Rightarrow \alpha + \beta + \gamma = \frac{-b}{a}$

$$-3 + 0 + 2 = \frac{-(a-1)}{1}$$

$$-1 = -a + 1 \Rightarrow -1 - 1 = -a$$

$$\therefore a = 2$$

Product of the roots $\alpha\beta\gamma = \frac{-d}{a}$

$$(-3)(0)(2) = \frac{-c}{1} \Rightarrow c = 0$$

$\therefore a = 2$ and $c = 0$

4. Find the discriminant of the quadratic equation $3x^2 - 5x + 2 = 0$ and hence write the nature of its roots.

Sol. Given quadratic equation

$$3x^2 - 5x + 2 = 0$$

Given quadratic equation compare with $ax^2 + bx + c = 0$

Here $a = 3; b = -5; c = 2$

Therefore, the discriminant

$$b^2 - 4ac = (-5)^2 - 4(3)(2)$$
$$= 25 - 24 = 1 > 0$$

$\therefore$ The quadratic equation has distinct and real roots.

5. Find the 11^{th} term of the A.P. : 15, 12, 9, ...

Sol. Here

$a_1 = 15; a_2 = 12$

$d = a_2 - a_1 = 12 - 15 = -3$

$a_n = a + (n-1)d$

$a_{11} = 15 + (11-1)(-3) = 15 - 30$

$a_{11} = -15$

6. If A = {1, 2, 3}, B = {3, 4, 5}, then find A – B and B – A.

Sol. A – B = {1, 2, 3} – {3, 4, 5} = {1, 2}

B – A = {3, 4, 5} – {1, 2, 3} = {4, 5}

7. Write any two linear polynomials having one term and three terms.

Sol. Linear polynomial one term = 2x.

Linear polynomial three terms = x + y + z

SECTION – II

8. If A = {x : x is a factor of 12} and B = {x : x is a factor of 6}, then find A ∪ B and A ∩ B.

Sol. Given A = {x : x is a factor of 12}

= {1, 2, 3, 4, 6, 12}

B = {x : x is a factor of 6}

= {1, 2, 3, 6}

$A \cup B = \{1, 2, 3, 4, 6, 12\} \cup \{1, 2, 3, 6\}$
$= \{1, 2, 3, 4, 6, 12\}$
$A \cap B = \{1, 2, 3, 4, 6, 12\} \cap \{1, 2, 3, 6\}$
$= \{1, 2, 3, 6\}$

9. Find the roots of quadratic equation $x^2 + 4x + 3 = 0$ by "Completing Square method".

Sol. Given quadratic equation is
$x^2 + 4x + 3 = 0$
$x^2 + 4x = -3$
$x^2 + 2 \cdot x \cdot 2 = -3$
Now LHS is of the form $a^2 + 2ab$, where $b = 2$.
Adding $b^2 = 2^2$ on both sides, we get
$x^2 + 2(x)(2) + (2)^2 = -3 + (2)^2$
$(x + 2)^2 = -3 + 4$
$(x + 2)^2 = 1$
$x + 2 = \pm 1$
$x + 2 = 1$; $x + 2 = -1$
$x = 1 - 2 = -1$; $x = -1 - 2 = -3$
$\therefore -1, -3$ are the roots of the given Q.E.

10. For what value of 'm' in the following, $mx + 4y = 10$ and $9x + 12y = 30$ system of equations will have no solution ? Why ?

Sol. Given equations have no solutions.
They have no solution mean they are parallel.
Given equations compare with
$a_1x + b_1y + c_1 = 0$ and
$a_2x + b_2y + c_2 = 0$ are parallel

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ here given

$a_1 = m$, $b_1 = 4$, $c_1 = -10$
$a_2 = 9$, $b_2 = 12$, $c_2 = -30$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \Rightarrow \frac{m}{9} = \frac{4}{12} \Rightarrow m = 3$

$\therefore$ If $m = 3$ then the above system will have no solution.

11. Which term of the G.P. : $\sqrt{2}, 2, 2\sqrt{2}, 4$, ... is 32 ?

Sol. Given G.P. : $\sqrt{2}, 2, 2\sqrt{2}, 4$, ... is 32.
$a = \sqrt{2}$; $r = \frac{a_2}{a_1} = \frac{2}{\sqrt{2}} = \sqrt{2}$
Now $a_n = a \cdot r^{n-1} = 32$
$(\sqrt{2})(\sqrt{2})^{n-1} = 32$
$(\sqrt{2})^n = 32 \Rightarrow (\sqrt{2})^n = 2^5$
$\therefore n = 10$
So 10^{th} term of
G.P. : $\sqrt{2}, 2, 2\sqrt{2}, 4$, ... is 32.

12. If $x^2 + y^2 = 10xy$, prove that $2 \log (x + y) = \log x + \log y + 2 \log 2 + \log 3$.

Sol. Given equation $x^2 + y^2 = 10xy$
Adding 2xy on both sides
$x^2 + y^2 + 2xy = 10xy + 2xy$
$(x + y)^2 = 12xy$
Applying log on both sides
$\log (x + y)^2 = \log (12xy)$
$2\log (x + y) = \log 12 + \log x + \log y$
$\therefore 2 \log (x + y) = \log 4 + \log 3 + \log x + \log y$

Hence proved.

13. Shashanka said that $(x + 1)^2 = 2(x - 3)$ is a quadratic equation. Do you agree ?

Sol. $(x + 1)^2 = 2(x - 3)$
$x^2 + 1 + 2x = 2x - 6$
$x^2 + 1 + 2x - 2x + 6 = 0$
$x^2 + 7 = 0 \Rightarrow x^2\ 0.x + 7 = 0$
Yes, this is a quadratic equation.

SECTION – III

14. Use division algorithm to show that the square of any positive integer is of the form 5m or 5m + 1 or 5m + 4, where 'm' is a whole number.

Sol. $a = bq + r, 0 \leq r < b$
$b = 5$ so $r = 0, 1, 2, 3, 4$

Then 'a' can be of the forms

$5q + 0, 5q + 1, 5q + 2, 5q + 3, 5q + 4$

Case(i) When $a = 5q$

$a^2 = (5q)^2 = 5(5q^2) = 5m$

where $m = 5q^2 \in W$.

Case (ii) When $a = 5q + 1$

$a^2 = (5q + 1)^2$

$= 25q^2 + 10q + 1$

$= 5(5q^2 + 2q) + 1$

$= 5m + 1$ where $m = 5q^2 + 2q \in W$

Similarly,

Case (iii) $a^2 = (5q + 2)^2 = 5m + 4$

Case (iv) $a^2 = (5q + 3)^2 = 5m + 4$

Case (v) $a^2 = (5q + 4)^2 = 5m + 1$

So the square of any positive integer is of the form 5m or 5m + 1 or 5m + 4 where $n \in W$.

OR

Show that $\sqrt{5} - \sqrt{3}$ is an irrational number.

Sol. Suppose $\sqrt{5} - \sqrt{3}$ is not an irrational number. $\sqrt{5} - \sqrt{3}$ is a rational number.

Let $\sqrt{5} - \sqrt{3} = \frac{p}{q}$ where $q \neq 0$ and $p, q \in Z$ squaring on both sides

$$5 + 3 - 2\sqrt{15} = \frac{p^2}{q^2}$$

$$\sqrt{15} = \frac{8q^2 - p^2}{2q^2}$$

$\therefore p, q \in Z$ & $q \neq 0$

$8q^2 - p^2$ & $2q^2 \in Z$ and also $2q^2 \neq 0$.

So $\frac{8q^2 - p^2}{2q^2}$ is a rational number.

but $\sqrt{15}$ is an irrational number.

An irrational number never be equal to a rational number.

So our supposition that $\sqrt{5} - \sqrt{3}$ is not an irrational number is false.

$\therefore \sqrt{5} - \sqrt{3}$ is an irrational number.

15. Draw the graph for the polynomial $p(x) = x^2 + 3x - 4$ and hence find the zeroes from the graph.

Sol. $p(x) = x^2 + 3x - 4$

x	–4	–3	–2	–1	0	1	2
x^2	16	9	4	1	0	1	4
3x	–12	–9	–6	–3	0	3	6
–4	–4	–4	–4	–4	–4	–4	–4
y	0	–4	–6	–6	–4	0	6
(x, y)	(–4, 0)	(–3, –4)	(–2, –6)	(–1, –6)	(0, –4)	(1, 0)	(2, 6)

Check Method :

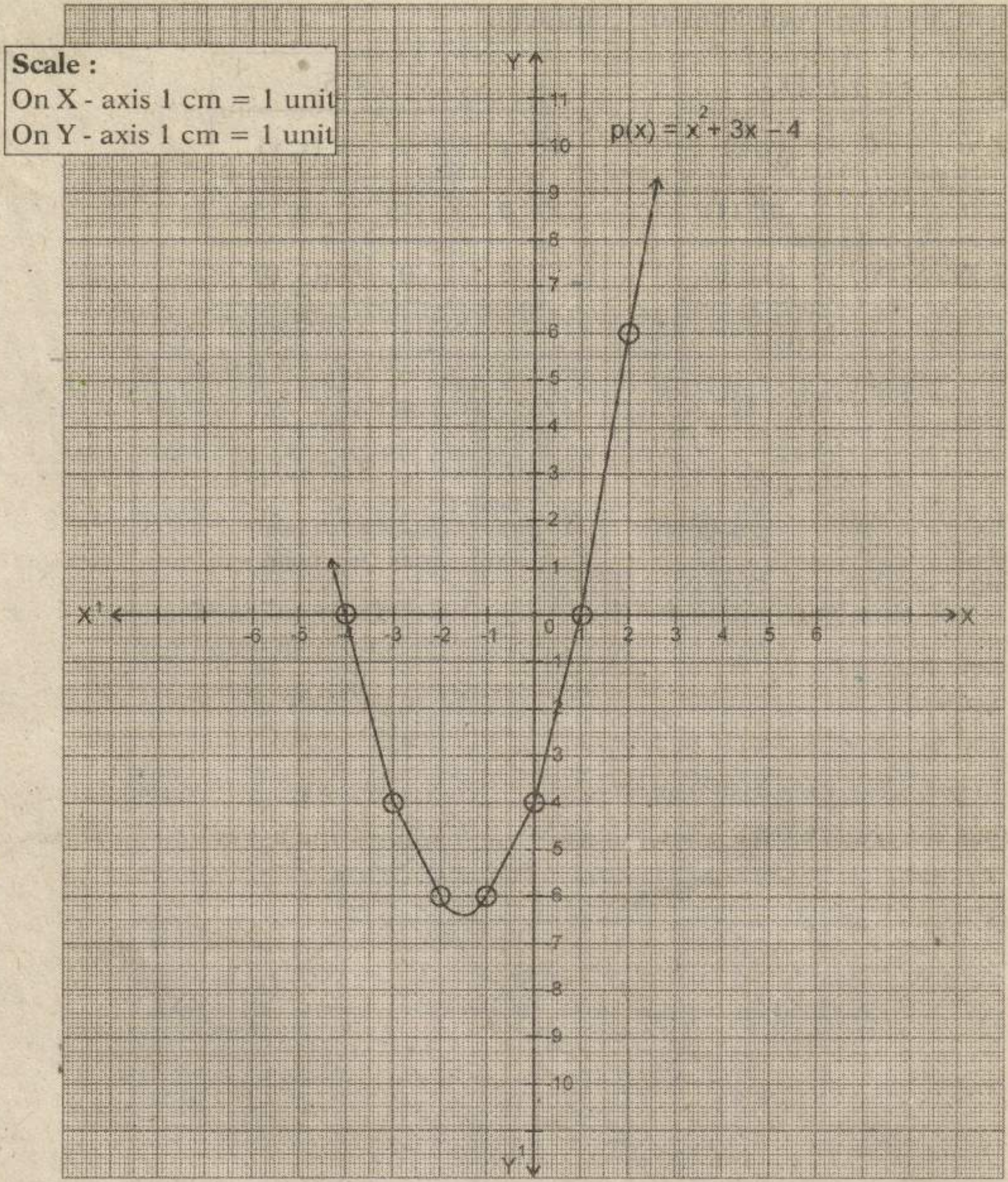
$p(x) = x^2 + 3x - 4$

$= x^2 + 4x - x - 4$

$= x(x + 4) - 1(x + 4)$

$p(x) = (x + 4)(x - 1)$

$p(x) = 0 \Rightarrow x = -4$ & $x = 1$



∴ Zeroes of polynomial are –4 and 1.

OR

Draw the graph of x + y = 11 and x – y = 5. Find the solution of the pair of linear equations.

Sol. Given x + y = 11 and x – y = 5

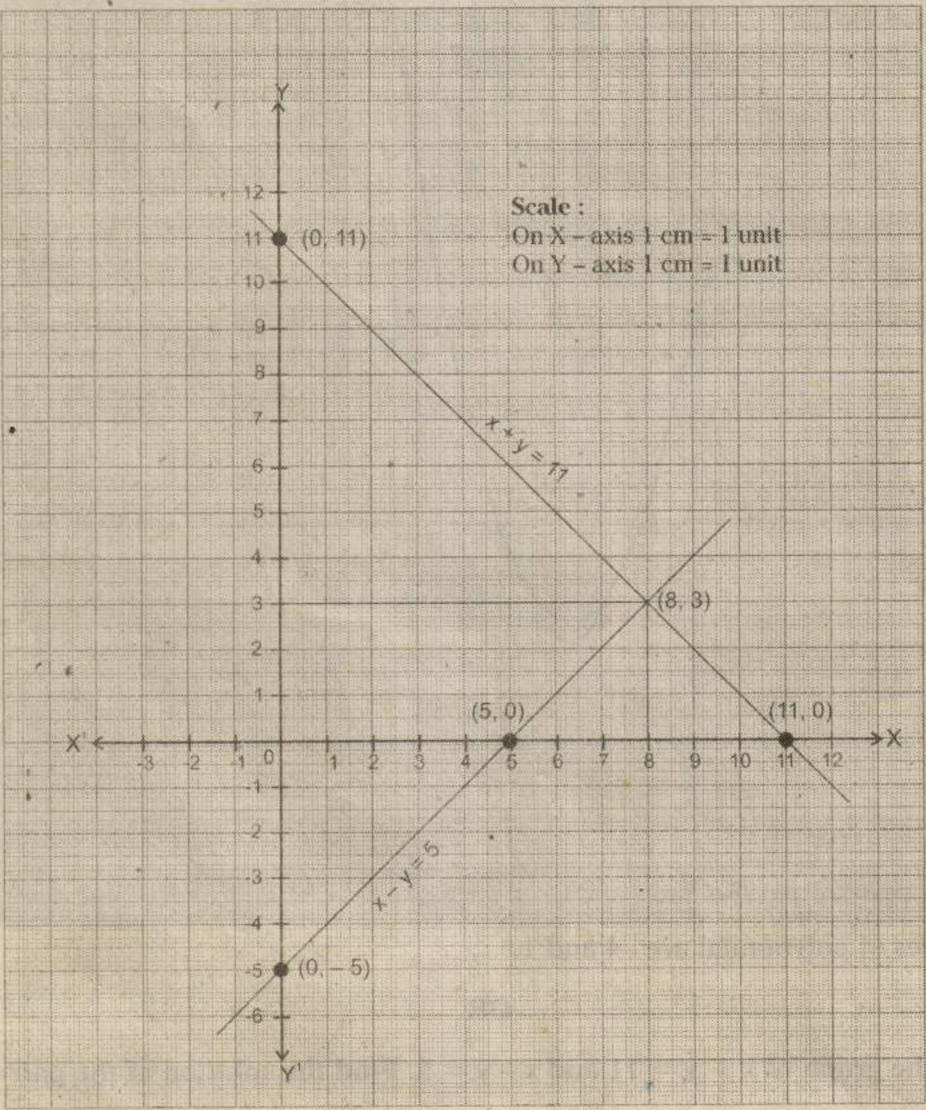
$a_1x + b_1y + c_1 = 0; a_2x + b_2y + c_2 = 0$

$\frac{a_1}{a_2} = \frac{1}{1}; \frac{b_1}{b_2} = \frac{1}{-1} \therefore \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Hence the linear equations are consistent.

x + y = 11		
x	y	(x, y)
0	11	(0, 11)
11	0	(11, 0)

x − y = 5		
x	y	(x, y)
0	−5	(0, −5)
5	0	(5, 0)



∴ The lines intersecting at a point (8, 3)

Check Method : Intersect at a point (8, 3)

x + y = 11	x − y = 5
8 + 3 = 11	8 − 3 = 5
11 = 11	5 = 5

16. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Sol. The distance travelled = 360 km

Let the speed of the train = x kmph

Time taken to complete a journey

$$= \frac{\text{distance}}{\text{speed}}$$

By problem $\frac{360}{x} - \frac{360}{x+5} = 1$

$$360\left(\frac{1}{x} - \frac{1}{x+5}\right) = 1$$

$$360\left(\frac{x+5-x}{x(x+5)}\right) = 1$$

$$\frac{5}{x^2+5x} = \frac{1}{360}$$

$x^2 + 5x = 1800$

$x^2 + 5x - 1800 = 0$

$x^2 + 45x - 40x - 1800 = 0$

$x(x + 45) - 40(x + 45) = 0$

$(x + 45)(x - 40) = 0$

$x + 45 = 0$ (or) $x - 40 = 0$

$x = -45$ or $x = +40$

But x can't be negative.

$\therefore$ The speed of the train = 40 kmph

OR

Find the sum of all the integers between 1 to 50 which are not divisible by 3.

Sol. [1, 4, 7, 10, ... 49] + [2, 5, 8, ... 50]

Case (i) : [1, 4, 7, 10, ... 49]

Here a = 1, d = 3

$a_n = a + (n-1)d$

$49 = 1 + (n-1)3 \Rightarrow 49 = 3n - 2$

$49 + 2 = 51 = 3n$

$$n = \frac{51}{3} = 17$$

$$S_{n(1)} = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{17}{2}[2(1) + (17-1)3]$$

$$= \frac{17}{2}[2 + 48]$$

$$= \frac{17}{2} \times 50 = 425$$

Case (ii) : [2, 5, 8, ...50]

a = 2, d = 3

$a_n = a + (n-1)d$

$50 = 2 + (n-1)3 = 3n - 1 \Rightarrow \frac{51}{3} = n$

$\therefore n = 17$

$$S_{n(2)} = \frac{17}{2}[2(2) + (17-1)3]$$

$$= \frac{17}{2}[4 + 48] = \frac{17}{2}[52] = 442$$

Adding case (i) & case (ii)

$S_n = 425 + 442 = 867.$

17. Find the area of a rhombus ABCD, whose vertices taken in order, are A (–1, 1), B(1, –2), C(3, 1) and D(1, 4).

Sol. Area of a rhombus

$$= \frac{1}{2} \times (\text{Product of its diagonals})$$

Let the vertices be $= \frac{1}{2} \times AC \times BD$

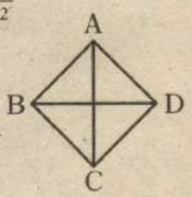
A (–1, 1), B (1, –2), C (3, 1), D (1, 4)

Distance formula

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(1+1)^2 + (-2-1)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$



$$BC = \sqrt{(3-1)^2 + (1+2)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$

$$CD = \sqrt{(1-3)^2 + (4-1)^2}$$

$$= \sqrt{4+9} = \sqrt{13}$$

$AD = \sqrt{(1+1)^2 + (4-1)^2}$

$= \sqrt{4+9} = \sqrt{13}$

$AC = \sqrt{(3+1)^2 + (1-1)^2}$

$= \sqrt{4^2 + 0^2} = 4$

$BD = \sqrt{(1-1)^2 + (4+2)^2}$

$= \sqrt{0 + 6^2} = 6$

In ▱ ABCD, AB = BC = CD = AD

[from sides are equal]

Hence ▱ ABCD is a rhombus.

$\therefore$ Area of Rhombus $= \frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 4 \times 6$

$= 12$ sq. units.

OR

If $A = \{x : x$ is a prime less than $20\}$ and $B = \{x : x$ is a whole number less than $10\}$, then verify $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Sol. Given

$A = \{x : x$ is a prime less than $20\}$

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

$n(A) = 8$

$B = \{x : x$ is a whole number less than $10\}$

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$n(B) = 10$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ – (1)

$A \cap B = \{2, 3, 5, 7, 11, 13, 17, 19\} \cap \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$= \{2, 3, 5, 7\}$

$n(A \cap B) = 4$

From (1) $n(A \cup B) = 8 + 10 - 4 = 14$.

PART - B

1) A 2) C 3) C 4) A 5) B 6) B 7) C 8) C 9) D 10) C