CLASS - X

TELANGANA

A GRADE MODEL PAPER 3

MATHEMATICS : PAPER - I

JUNE 2018

Time : 2.45 Hours] **Parts - A and B** [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the question paper.
2. Answer **all** Questions under **Part – A** on a separate answer book.
3. Write the answers to the Questions under **Part – B** on the Question paper itself and attach it to the answer book of **Part – A**.

Time : 2 hrs 15 min.] **PART - A** **[Marks : 35**

Note :

i) Answer **all** the questions from the given **three** sections I, II and III of **Part – A**.

ii) In section – III, every question has internal choice. Answer **any one** alternative.

SECTION – I

(Marks : 7 × 1 = 7)

Note : i) Answer **all** the following questions.

ii) Each question carries **1** mark.

1. Find the values of k for which the quadratic equation $4x^2 + 5kx + 25 = 0$ has equal roots.
2. List all its subsets of the Set A = {x, y, z}.
3. Find the value of $\log_{\sqrt{2}} 128$.
4. Whether $\frac{1}{2}$ and 1 are zeroes of the polynomial $p(x) = 2x^2 - 3x + 1$ or not ? Justify.
5. For the A.P.; – 3, – 7, – 11, ... ; can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}.
6. Whether the following pair of linear equations are parallel ? Justify.

   $6x - 4y + 10 = 0,\ 3x - 2y + 6 = 0$
7. What is the other end of the diameter of the Circle, whose centre is (1, 2) and one end point of the diameter is (3, 4) ?

SECTION – II

(Marks : 6 × 2 = 12)

Note : i) Answer **all** the following questions.

ii) Each question carries **2** marks.

8. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8, 10}, then represent the Venn diagram of A – B.
9. Find the coordinates of point which divides the segment joining (2, 3) and (– 4, 0) in 1 : 2.
10. If one of the zeroes of the cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ is zero, then find the product of other two zeroes of p(x). $(a \neq 0)$
11. Without calculating the roots of $x^2 - 5x + 6 = 0$, explain the nature of roots.
12. Lalitha says that HCF and LCM of the numbers 80 and 60 are 20 and 120 respectively. Do you agree with her ? Justify.
13. In a rangoli design of thirteen rows, every row increases its previous row by two dots and first row contains 5 dots, then how many total dots are in the design ?

SECTION – III

(Marks : 4 × 4 = 16)

Note : i) Answer **all** the following questions.

ii) In this section, every question has internal choice.

iii) Answer **any one** alternative.

iv) Each question carries **4** marks.

14. If A = {x : x is a prime and x < 10}, B = {x : x is a factor of 6}, then find $A \cap B$, $A \cup B$ and A – B.

OR

Find the sum of the integers between 100 and 500 that are divisible by 9.

15. If a number when increased by 12, equals 160 times of its reciprocal, then find the numbers.

OR

Solve : $\dfrac{5}{x-1} + \dfrac{1}{y-2} = 2, \quad \dfrac{6}{x-1} - \dfrac{1}{y-2} = 1$

16. Prove that $\sqrt{2} + \sqrt{11}$ is an irrational number.

OR

Show that the points A(–1, –2), B(4, 3), C(2, 5) and D(– 3, 0) in that order form a rectangle.

17. Draw the graph of the polynomial $p(x) = x^2 + x - 2$ on the graph paper. Find its zeroes from the graph.

OR

Solve the following pair of linear equations by graph method.

$2x + y = 6$ and $2x - y + 2 = 0$

Time : 30 Mts.] **PART - B** **[Marks : 5**

Note :

(i) Write the CAPITAL LETTERS (A, B, C, D) showing the correct answer for the following questions in the brackets provided against them.

(ii) Answer **all** the questions.

(iii) Each question carries ½ mark.

(iv) Answers are to be written in the question paper only.

(v) Marks will **not** be awarded in any case of overwriting, rewriting or erased answers.

1. If set A and B are disjoint sets and $n(A) = 6$, $n(B) = 5$, then $n(A \cup B) =$ [ ]

A) 11 B) 6 C) 5 D) 1

2. The decimal expansion of 0.225 in its rational form is [ ]

A) 225 B) $\frac{225}{10^2}$ C) $\frac{225}{10^4}$ D) $\frac{9}{40}$

3. If $p(x) = x^2 - 4x + 5$, then the value of $p(1)$ is [ ]

A) – 1 B) 0 C) 1 D) 2

4. If $2x + 3y = 8$ and $4x + py = 16$ has infinite solutions, then $p =$ .... [ ]

A) 8 B) 6 C) 10 D) 16

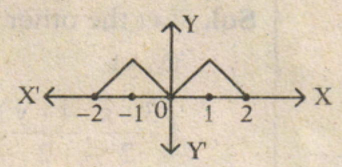
5. From the graph, the zeroes of the polynomial are .... [ ]

A) – 2

B) 0

C) 2

D) All the above



6. Every even positive integer can be written in the form of .... [ ]

A) $2p + 1$ $(p \in Z^+)$ B) $2p - 1$ $(p \in Z^+)$ C) $2p$ $(p \in Z^+)$ D) $3p$ $(p \in Z^+)$

7. Which of the following is true ? [ ]

A) $\phi = 0$ B) $\phi = \{\ \}$ C) $\phi = \{0\}$ D) Both A and C

8. Sum of the first 10 natural numbers is [ ]

A) $\frac{10 \times 9}{2}$ B) $\frac{10 \times 10}{2}$ C) $\frac{10 \times 11}{2}$ D) Both A and B

9. What does 'r' represents in the general term of GP, $a_n = ar^{n-1}$ [ ]

A) Radius B) Common ratio

C) Common difference D) Common multiple

10. If slope of a line is "1", then the angle between the line and X - axis is [ ]

A) 45° B) 30° C) 60° D) 90°

SOLUTIONS

PART - A

SECTION – I

1. Find the values of k for which the quadratic equation $4x^2 + 5kx + 25 = 0$ has equal roots.

Sol. $a = 4, b = 5k, c = 25$

Q. E. has equal roots $\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (5k)^2 - 4(4)(25) = 0$

$\Rightarrow k^2 = \frac{400}{25} = 16 \Rightarrow k = \pm 4.$

2. List all its subsets of the Set A = {x, y, z}.

Sol. A = {x, y, z}

Subsets ϕ, {x}, {y}, {z}, {x, y}, {y, z}, {z, x}, A

3. Find the value of $\log_{\sqrt{2}} 128$.

Sol. $\log_{\sqrt{2}} 128 = x$ (Let)

$(\sqrt{2})^x = 128 = 2^7 = (\sqrt{2})^{14}$

Bases are equal, equate powers

$x = 14$

$\therefore \log_{\sqrt{2}} 128 = 14$

4. Whether $\frac{1}{2}$ and 1 are zeroes of the polynomial $p(x) = 2x^2 - 3x + 1$ or not ? Justify.

Sol. $p(x) = 2x^2 - 3x + 1$

$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$

$= \frac{1}{2} - \frac{3}{2} + 1 = 0$

$p(1) = 2(1)^2 - 3(1) + 1 = 2 - 3 + 1 = 0$

$\therefore \frac{1}{2}$ and 1 are zeroes of the polynomial p(x).

5. For the A.P.; – 3, – 7, – 11, ... ; can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}.

Sol. Given A.P. – 3, –7, –11, ...

$a = -3, d = -7 - (-3) = -4$

$a_{30} - a_{20} = (a + 29d) - (a + 19d)$

$= a + 29d - a - 19d = 10d$

$= 10(-4) = -40$

6. Whether the following pair of linear equations are parallel ? Justify.

$6x - 4y + 10 = 0, \; 3x - 2y + 6 = 0$

Sol. $6x - 4y + 10 = 0, 3x - 2y + 6 = 0$

$\frac{a_1}{a_2} = \frac{6}{3} = 2, \; \frac{b_1}{b_2} = \frac{-4}{-2} = 2, \; \frac{c_1}{c_2} = \frac{10}{6} = \frac{5}{3}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So the lines are parallel.

7. What is the other end of the diameter of the Circle, whose centre is (1, 2) and one end point of the diameter is (3, 4) ?

Sol. Let the other end point of diameter be (x, y).

$\left(\frac{3+x}{2}, \frac{4+y}{2}\right) = (1, 2)$

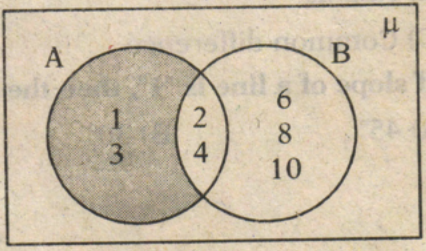
$\frac{3+x}{2} = 1$	$\frac{4+y}{2} = 2$
$3 + x = 2$	$4 + y = 4$
$x = 2 - 3 = -1$	$y = 4 - 4 = 0$

$\therefore$ The required point is (– 1, 0).

SECTION – II

8. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8, 10}, then represent the Venn diagram of A – B.

Sol.



A – B = {1, 3}

9. Find the coordinates of point which divides the segment joining (2, 3) and (– 4, 0) in 1 : 2.

Sol. (2, 3) (– 4, 0) 1 : 2

$x_1 = 2$ $x_2 = -4$ $m_1 = 1$

$y_1 = 3$ $y_2 = 0$ $m_2 = 2$

Required point

$$= \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$= \left(\frac{1(-4) + 2(2)}{1+2}, \frac{1(0) + 2(3)}{1+2}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{0+6}{3}\right)$$

$$= \left(\frac{0}{3}, \frac{6}{3}\right) = (0, 2)$$

10. If one of the zeroes of the cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$ is zero, then find the product of other two zeroes of p(x). $(a \neq 0)$

Sol. $p(x) = ax^3 + bx^2 + cx + d$

Let the zeroes of the polynomial be α, β and γ, given $\gamma = 0$.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta = \frac{c}{a} \quad [\because \gamma = 0]$$

11. Without calculating the roots of $x^2 - 5x + 6 = 0$, explain the nature of roots.

Sol. $x^2 - 5x + 6 = 0$

$a = 1, b = -5, c = 6.$

Discriminent (D) $= b^2 - 4ac$

$= (-5)^2 - 4(1)(6)$

$= 25 - 24 = 1$

D > 0, so roots are real and distinct.

12. Lalitha says that HCF and LCM of the numbers 80 and 60 are 20 and 120 respectively. Do you agree with her ? Justify.

Sol. We know that LCM × HCF

= Product of the two numbers

Product of the two numbers

$= 80 \times 60 = 4800$

$HCF \times LCM = 20 \times 120 = 2400$

Since they are not equal, I don't agree with her.

13. In a rangoli design of thirteen rows, every row increases its previous row by two dots and first row contains 5 dots, then how many total dots are in the design ?

Sol. 5, 7, 9, 11, ... are in A.P.

$a = 5, d = 2, n = 13$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{13}{2}[2(5) + (13-1)2]$$

$$= \frac{13}{2}(10 + 24) = \frac{13}{2} \times 34 = 221$$

SECTION – III

14. If A = {x : x is a prime and x < 10}, B = {x : x is a factor of 6}, then find $A \cap B$, $A \cup B$ and A – B.

Sol. A = {2, 3, 5, 7}; B = {1, 2, 3, 6}

$A \cap B = \{2, 3\}$

$A \cup B = \{1, 2, 3, 5, 6, 7\}$

$A - B = \{5, 7\}$

OR

Find the sum of the integers between 100 and 500 that are divisible by 9.

Sol. Integers between 100 and 500 which are divisible by 9 are

108, 117, 126, ..., 495 are in A.P.

$a = 108, d = 117 - 108 = 9$

$a_n = a + (n-1)d$

$495 = 108 + (n-1)9$

$$n - 1 = \frac{495 - 108}{9} = \frac{387}{9} = 43$$

$n = 43 + 1 = 44$

$$S_n = \frac{n}{2}[a + l]$$

$$S_{44} = \frac{44}{2}[108 + 495]$$

$= 22 \times 603 = 13,266$

15. If a number when increased by 12, equals 160 times of its reciprocal, then find the numbers.

Sol. Let the number be 'x'

$x + 12 = 160 \times \frac{1}{x}$

$x^2 + 12x - 160 = 0$, Quadratic equation

$x^2 + 20x - 8x - 160 = 0$

$x(x + 20) - 8(x + 20) = 0$

$(x + 20)(x - 8) = 0$

$x = -20$ or $x = 8$

$\therefore$ Required number is -20 or 8.

OR

Solve :

$$\frac{5}{x-1}+\frac{1}{y-2} = 2, \quad \frac{6}{x-1}-\frac{1}{y-2} = 1$$

Sol. Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$

$5a + b = 2$

on adding $6a - b = 1$

$11a = 3$

$\Rightarrow a = \frac{3}{11}$

$5\left(\frac{3}{11}\right) + b = 2$

$b = 2 - \frac{15}{11} = \frac{22-15}{11} \Rightarrow b = \frac{7}{11}$

$\frac{1}{x-1} = \frac{3}{11}$	$\frac{1}{y-2} = \frac{7}{11}$
$3x - 3 = 11$	$7y - 14 = 11$
$3x = 14$	$7y = 25$
$x = \frac{14}{3}$	$y = \frac{25}{7}$

16. Prove that $\sqrt{2}+\sqrt{11}$ is an irrational number.

Sol. Suppose $\sqrt{2}+\sqrt{11}$ is not an irrational number.

$\sqrt{2}+\sqrt{11}$ a rational number.

Let $\sqrt{2}+\sqrt{11} = \frac{p}{q}$,

where $q \neq 0$ and $p, q \in Z$.

Squaring on both sides

$2 + 11 + 2\sqrt{22} = \frac{p^2}{q^2}$

$\sqrt{22} = \frac{p^2 - 13q^2}{2q^2}$

$p^2 - 13q^2$ & $2q^2 \in Z$ and also $2q^2 \neq 0$

$[\because p, q \in Z \;\&\; q \neq 0]$

So $\frac{p^2-13q^2}{2q^2}$ is a rational number.

But $\sqrt{22}$ is an irrational number.

An irrational number never become equal to a rational number.

So our supposition that $\sqrt{2}+\sqrt{11}$ is not an irrational number is false.

$\therefore \sqrt{2}+\sqrt{11}$ is an irrational number.

OR

Show that the points A(–1, –2), B(4, 3), C(2, 5) and D(– 3, 0) in that order form a rectangle.

Sol. Given points are A (– 1, – 2), B (4, 3) C (2, 5), D (– 3, 0)

Mid point of AC $= \left(\frac{-1+2}{2}, \frac{-2+5}{2}\right)$

$= \left(\frac{1}{2}, \frac{3}{2}\right)$

Mid point of BD $= \left(\frac{4-3}{2}, \frac{3+0}{2}\right)$

$= \left(\frac{1}{2}, \frac{3}{2}\right)$

Length of diagonal

AC $= \sqrt{(2+1)^2 + (5+2)^2}$

$= \sqrt{3^2 + 7^2} = \sqrt{9+49} = \sqrt{58}$ units.

Length of diagonal

BD $= \sqrt{(-3-4)^2 + (0-3)^2}$

$= \sqrt{(-7)^2 + (-3)^2}$

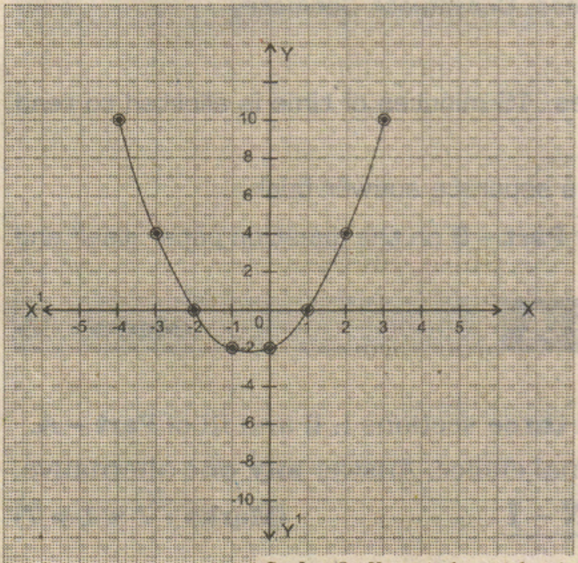
$= \sqrt{49+9} = \sqrt{58}$ units.

Diagonal are equal and bisect each other. So given vertices form a rectangle.

17. Draw the graph of the polynomial $p(x) = x^2 + x - 2$ on the graph paper. Find its zeroes from the graph.

Sol. $p(x) = x^2 + x - 2$

x	–4	–3	–2	–1	0	1	2	3
x^2	16	9	4	1	0	1	4	9
x	–4	–3	–2	–1	0	1	2	3
–2	–2	–2	–2	–2	–2	–2	–2	–2
y	10	4	0	–2	–2	0	4	10
(x, y)	(–4,10)	(–3,4)	(–2,0)	(–1,–2)	(0,–2)	(1,0)	(2,4)	(3,10)



Scale : On X – axis 1 cm = 1 unit
On Y – axis 1 cm = 2 units

Zeros of the given polynomial are – 2, 1.

OR

Solve the following pair of linear equations by graph method.

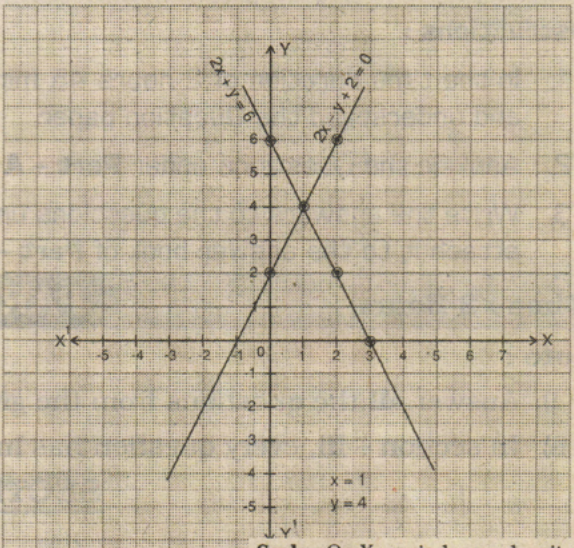
$2x + y = 6$ and $2x - y + 2 = 0$

Sol. $2x + y = 6$

x	0	1	2	3
y	6	4	2	0
(x, y)	(0,6)	(1,4)	(2,2)	(3,0)

$2x - y + 2 = 0$

x	0	1	2
y	2	4	6
(x, y)	(0,2)	(1,4)	(2,6)



Scale : On X – axis 1 cm = 1 unit
On Y – axis 1 cm = 1 unit

Point of intersection is (1, 4).

$\therefore x = 1; y = 4$

PART - B

1) A 2) D 3) D 4) B 5) D 6) C 7) B 8) C 9) B 10) A

✣ ✣ ✣ ✣ ✣