CLASS - X

TELANGANA

A GRADE MODEL PAPER 5

MATHEMATICS : PAPER - I

JUNE 2017

Time : 2 hours 45 min.] **Parts - A and B** [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. *In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.*
2. *Answer the Questions under **'Part – A'** on a separate answer book.*
3. *Write the answers to the Questions under **'Part – B'** on the Question paper itself and attach it to the answer book of **'Part – A'**.*

Time : 2 Hours] **PART - A** **[Marks : 35**

Note :

1. *Answer **all** the questions from the given **three** sections – I, II and III of **Part – A**.*
2. *In section – III, every question has internal choice. Answer **anyone** alternative.*

SECTION – I

(Marks : 7 × 1 = 7)

Note : (i) Answer **all** the following questions.

(ii) Each question carries **1** mark.

1. Find the HCF and LCM of 90, 144 by prime factorisation method.
2. Is $\log_3 81$ rational or irrational ? Justify your answer.
3. If A = {1, 2, 3, 5}, B = {3, 4, 5, 6}, find $A \cap B$.
4. Find the zeroes of the polynomial $p(x) = x^2 - 4$.
5. Write the condition for the pair of linear equations in two variables to be parallel lines.
6. Write the nature of the roots of the quadratic equation $x^2 - 8x + 16 = 0$.
7. The nth term of an A.P. is 6n + 2. Find the common difference. ($x \in N$)

SECTION – II

(Marks : 6 × 2 = 12)

Note : (i) Answer **all** the following questions.

(ii) Each question carries **2** marks.

8. Prove that $2 + \sqrt{3}$ is irrational.
9. If $A = \{x : x \in N, x < 10\}$, $B = \{x : x \text{ is a prime number and } x < 10\}$, then show that $A - B \neq B - A$ with the help of Venn – diagram.

10. Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ and verify the division lemma.

11. Is it possible to design a rectangular garden, whose length is twice of its breadth and area is 200 m^2 ? If so, find its length and breadth.

12. Find the value of 'k', so that $k + 2$, $4k - 6$ and $3k - 2$ are the three consecutive terms of an A.P.

13. Determine 'x', if the slope of the line joining the two points (4, x), (7, 2) is $\frac{8}{3}$.

SECTION – III

(Marks : 4 × 4 = 16)

Note : (i) Answer **all** the following questions.

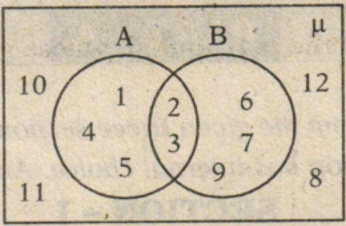
(ii) In this section, every question has internal choice.

(iii) Answer **anyone** alternative.

(iv) Each question carries **4** marks.

14. If $x^2 + y^2 = 27xy$, then show that $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}[\log x + \log y]$.

OR



Using the Venn diagram, verify $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

15. If -4 is a common root for the quadratic equations $2x^2 + px + 8 = 0$ and $p(x^2 + x) + k = 0$, then find the value of 'k'.

OR

The area of the triangle is 18 sq. units, whose vertices are (3, 4), (–3, –2) and (p, –1); then find the value of 'p'.

16. The sum of 5th and 9th terms of A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.

OR

Which term of G.P.: 3, 9, 27, ... is 2187 ?

17. Draw the graph of $p(x) = x^2 - 2x - 8$ and find the zeroes of the polynomial from it.

OR

Show that the following pair of equations are consistent and solve them graphically.

$x + 3y = 6,$

$2x - 3y = 12$

Time : 30 Minutes]	PART - B	[Marks : 5

Instructions :

*(i) Write the answers to the questions in this **Part-B** on the Question paper itself and attach it to the answer book of **Part-A**.*

*(ii) Answer **all** the questions.*

(iii) Each question carries ½ mark.

(iv) Answers are to be written in question paper only.

*(v) Marks will **not** be awarded in any case of over - writing, rewriting or erased answers.*

I. Write the **CAPITAL LETTERS (A,B,C,D)** showing the correct answer for the following questions in the brackets provided against them. ***(Marks : 10 × ½ = 5)***

1. **In the rational form of a terminating decimal number prime factor of the denominator is ....** [ ]
 (A) only 2 (B) only 5 (C) 2 or 5 only (D) Any prime

2. **$\log_{10}2 + \log_{10}5$ value = ...** [ ]
 (A) 1 (B) 2 (C) 5 (D) 10

3. **If $A \subset B$, then $A \cap B$ = ...** [ ]
 (A) A (B) B (C) $\varnothing$ (D) μ

4. **The number of subsets of a set is 16, then the set has .... elements.** [ ]
 (A) 1 (B) 2 (C) 3 (D) 4

5. **The number of zeros of the polynomial, whose graph is given below.** [ ]

 (A) 0
 (B) 1
 (C) 2
 (D) 3

6. **The value of x, which satisfies $2(x-1)-(1-x) = 2x + 3$** [ ]
 (A) 2 (B) 4 (C) 6 (D) 8

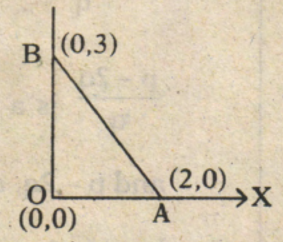
7. **In a quadratic equation $ax^2 + bx + c = 0$, if $b^2 - 4ac > 0$, then their roots are ....** [ ]
 (A) real and distinct (B) real and equal
 (C) imaginary (D) None

8. **The sum of first 100 natural numbers is ....** [ ]
 (A) 4050 (B) 4500 (C) 5500 (D) 5050

9. **If a, b, c are in G.P., then b = ...** [ ]
 (A) ac (B) $\sqrt{ac}$ (C) $\frac{a+c}{2}$ (D) a^2c^2

10. **The area of the triangle BOA is .... sq. units.** [ ]

 (A) 1
 (B) 2
 (C) 3
 (D) 4



SOLUTIONS

PART - A

SECTION – I

1. Find the HCF and LCM of 90, 144 by prime factorisation method.

Sol. $90 = 2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

$\text{H.C.F} = 2 \times 3^2 = 18$

$\text{L.C.M} = 2^4 \times 3^2 \times 5 = 720$

2. Is $\log_3 81$ rational or irrational ? Justify your answer.

Sol. $\log_3 81 = \log_3 3^4 = 4 \log_3 3 = 4$

So $\log_3 81$ is a rational number.

3. If A = {1, 2, 3, 5}, B = {3, 4, 5, 6}, find $A \cap B$.

Sol. $A = \{1, 2, 3, 5\}$, $B = \{3, 4, 5, 6\}$

$A \cap B = \{1, 2, 3, 5\} \cap \{3, 4, 5, 6\}$

$= \{3, 5\}$

4. Find the zeroes of the polynomial $p(x) = x^2 - 4$.

Sol. $p(x) = x^2 - 4$

$x^2 - 4 = 0$

$x^2 = 4$

$\therefore x = \sqrt{4} = \pm 2$

5. Write the condition for the pair of linear equations in two variables to be parallel lines.

Sol. Condition for the lines

$a_1x + b_1y + c_1 = 0$ and

$a_2x + b_2y + c_2 = 0$ to be

parallel lines is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

6. Write the nature of the roots of the quadratic equation $x^2 - 8x + 16 = 0$.

Sol. $x^2 - 8x + 16 = 0$

Discriminent (D) $= b^2 - 4ac$

$= (-8)^2 - 4(1)(16)$

$= 64 - 64 = 0$

Since D = 0, roots are real and equal.

7. The nth term of an A.P. is 6n + 2. Find the common difference. ($x \in N$)

Sol. $a_n = 6n + 2$

$a_1 = 6(1) + 2 = 8$

$a_2 = 6(2) + 2 = 14$

Common difference (d) $= a_2 - a_1$

$= 14 - 8$

$= 6$

SECTION – II

8. Prove that $2 + \sqrt{3}$ is irrational.

Sol. Suppose $2 + \sqrt{3}$ is not an irrational number.

$2 + \sqrt{3}$ is a rational number.

Let $2 + \sqrt{3} = \frac{p}{q}$ where $q \neq 0$ and $p, q \in Z$

$\sqrt{3} = \frac{p}{q} - 2 = \frac{p - 2q}{q}$

$\frac{p - 2q}{q}$ is a rational number as $q \neq 0$ and $p - 2q, q \in Z$.

but $\sqrt{3}$ is an irrational number

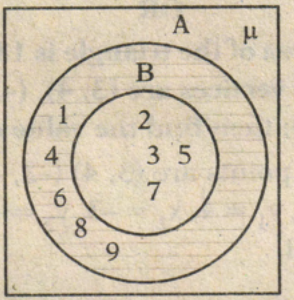
An irrational number never become equal to a rational number. So our supposition that $2 + \sqrt{3}$ is not an irrational number is false.

$\therefore 2 + \sqrt{3}$ is an irrational number.

9. If A = {x : x ∈ N, x < 10}, B = {x : x is a prime number and x < 10}, then show that A – B ≠ B – A with the help of Venn-diagram.

Sol. A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

B = {2, 3, 5, 7}



Since $B \subset A$, $B - A = \phi$

$\therefore A - B \neq B - A$

10. Divide $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ and verify the division lemma.

Sol. $x^2 - 2)\, x^3 - 3x^2 + 5x - 3\, (x - 3$

$x^3 \quad - 2x$

$(-) \quad (+)$

$- 3x^2 + 7x - 3$

$- 3x^2 \quad + 6$

$(+) \quad (-)$

$7x - 9$

Verification : (Divisor × Quotient) + Remainder = Dividend

$(x^2 - 2)(x - 3) + (7x - 9)$

$= x^3 - 3x^2 + 5x - 3$

11. Is it possible to design a rectangular garden, whose length is twice of its breadth and area is 200 m²? If so, find its length and breadth.

Sol. Let breadth be "x" m

then length be "2x" m

Area = $2x^2$ m²

$2x^2 = 200$

$x^2 = 100$

$x = \sqrt{100} = 10$

$\therefore$ breadth = 10 m

length = 20 m

$\therefore$ It is possible to design such a rectangular garden.

12. Find the value of 'k', so that k + 2, 4k – 6 and 3k – 2 are the three consecutive terms of an A.P.

Sol. a_1, a_2, a_3 be any three consecutive terms of an A.P

$a_2 - a_1 = a_3 - a_2$

$(4k - 6) - (k + 2) = (3k - 2) - (4k - 6)$

$3k - 8 = -k + 4$

$4k = 12$

$k = \frac{12}{4} = 3$

13. Determine 'x', if the slope of the line joining the two points (4, x), (7, 2) is $\frac{8}{3}$.

Sol. Slope = $\frac{y_2 - y_1}{x_2 - x_1}$

$\frac{8}{3} = \frac{2 - x}{7 - 4}$

$8 = 2 - x$

$\therefore x = 2 - 8 = -6$

SECTION - III

14. If $x^2 + y^2 = 27xy$, then show that $\log\left(\frac{x-y}{5}\right) = \frac{1}{2}[\log x + \log y]$.

Sol. $x^2 + y^2 = 27xy$

Subtracting 2 xy from both sides

$x^2 + y^2 - 2xy = 27xy - 2xy$

$(x-y)^2 = 25xy$

$$\left(\frac{x-y}{5}\right)^2 = xy$$

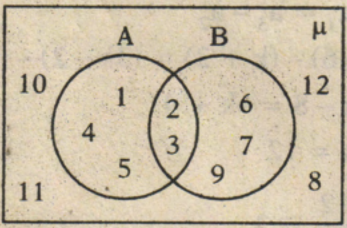
Applying 'log' on both sides

$$\log\left(\frac{x-y}{5}\right)^2 = \log xy$$

$$2\log\left(\frac{x-y}{5}\right) = \log x + \log y$$

$$\log\left(\frac{x-y}{5}\right) = \frac{1}{2}[\log x + \log y]$$

OR



Using the Venn diagram, verify $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

Sol. $A = \{1, 2, 3, 4, 5\}$ $\quad n(A) = 5$

$B = \{2, 3, 6, 7, 9\}$ $\quad n(B) = 5$

$A \cap B = \{2, 3\}$ $\quad n(A \cap B) = 2$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 9\}$

$n(A \cup B) = 8$

$n(A) + n(B) - n(A \cap B) = 5 + 5 - 2 = 8$

$n(A \cup B) = 8$

$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$

15. If –4 is a common root for the quadratic equations $2x^2 + px + 8 = 0$ and $p(x^2 + x) + k = 0$, then find the value of 'k'.

Sol. $2x^2 + px + 8 = 0$

$2(-4)^2 + p(-4) + 8 = 0$

$p(-4) = -40$

$$p = \frac{-40}{-4} = 10$$

$p(x^2 + x) + k = 0$

$10[(-4)^2 + (-4)] + k = 0$

$k = -120$

OR

The area of the triangle is 18 sq. units, whose vertices are (3, 4), (–3, –2) and (p, –1); then find the value of 'p'.

Sol. Given points are (3, 4) (–3, –2) (p, –1)

$x_1 = 3, y_1 = 4, x_2 = -3, y_2 = -2, x_3 = p, y_3 = -1$

$$\text{Area of triangle} = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$18 = \frac{1}{2}|3(-2+1) - 3(-1-4) + p(4+2)|$$

$\Rightarrow |-3 + 15 + 6p| = 36$

$\Rightarrow 6p + 12 = 36$ (or) $6p + 12 = -36$

$$p = \frac{36-12}{6} \qquad\Big|\qquad p = \frac{-36-12}{6}$$

$$p = \frac{24}{6} = 4 \qquad\Big|\qquad p = \frac{-48}{6} = -8$$

$\therefore p = 4$ or -8

16. The sum of 5th and 9th terms of A.P. is 72 and the sum of 7th and 12th terms is 97. Find the A.P.

Sol. $a_5 + a_9 = 72$

OR

Show that the following pair of equations are consistent and solve them graphically.

x + 3y = 6,

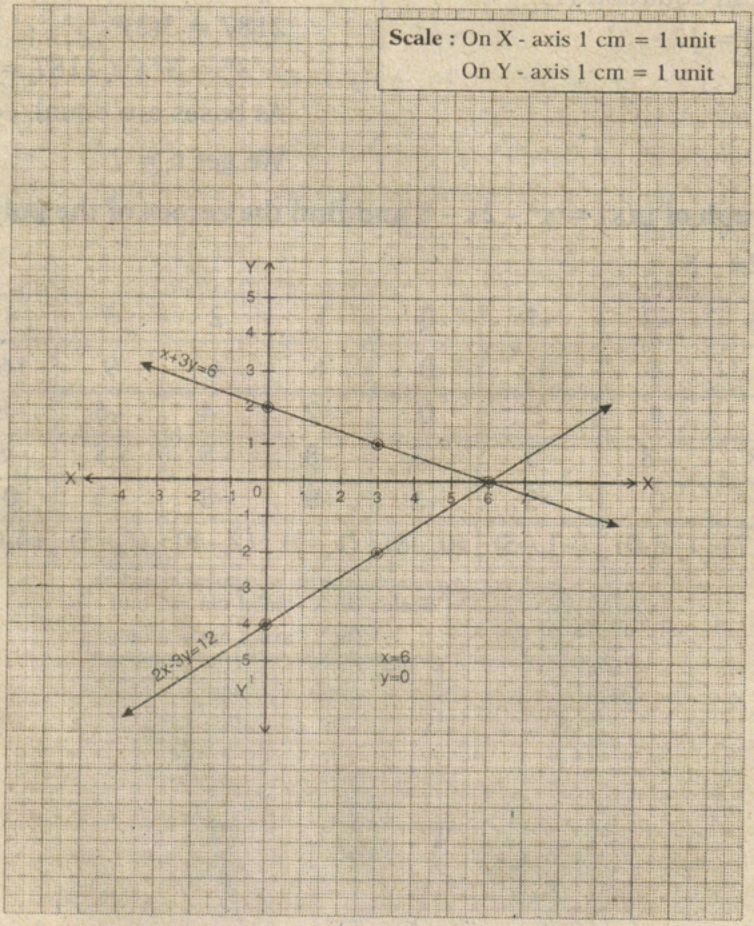
2x – 3y = 12

Sol. $x + 3y = 6$

x	0	6	3
y	2	0	1
(x,y)	(0,2)	(6,0)	(3,1)

$2x - 3y = 12$

x	0	6	3
y	–4	0	–2
(x,y)	(0,–4)	(6,0)	(3,–2)



Point of intersection is (6, 0)

Since the two lines are intersecting at one point, we can say that they are Consistent.

$\therefore x = 6, \quad y = 0$

PART - B

1) C 2) A 3) A 4) D 5) D 6) C 7) A 8) D 9) B 10) C

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