CLASS - X

TELANGANA

A GRADE

MODEL PAPER 7

MATHEMATICS : PAPER - I

JUNE 2016

Time : 2 hours 45 min.] | Parts - A and B | [Max. Marks : 40

Real Numbers, Sets, Polynomials, Pair of Linear Equations in Two Variables, Quadratic Equations, Progressions, Coordinate Geometry

Instructions :

1. In the time duration of 2 hours 45 minutes, 15 minutes of time is allotted to read and understand the Question paper.
2. Answer the Questions under **Part – A** on a separate answer book.
3. Write the answer to the questions under **Part – B** on the question paper itself and attach it to the answer book of **Part – A**.

Time : 2.15 Hours] | PART - A | [Max. Marks : 35

Note :

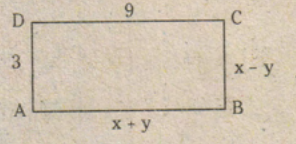
1. Answer **all** the questions from the given **three** sections I, II and III of **Part – A.**
2. In section – III, every question has internal choice. Answer **anyone** alternative.

SECTION – I

7 × 1 = 7

Note : (i) Answer **all** the following questions.
(ii) Each question carries **1** Mark.

1. If $p(x) = x^3 - 3x^2 + 2x - 3$ is a polynomial, then find the value of $p(1)$.
2. Find the value of k, if 2 is one of the roots of the quadratic equation $x^2 - kx + 6 = 0$.
3. $A = \{x : x \in N, x \text{ is a composite number and } x < 13\}$. Write set A in the roster form.
4. In a rectangle ABCD, AB = x + y, BC = x – y, CD = 9 and AD = 3. Find the values of x and y.



5. Write any two irrational numbers lying between 3 and 4.
6. If the slope of the line passing through the two points (2, 5) and (5, 8) is represented by $\tan\theta$; (where $0° < \theta < 90°$) in trigonometry, then find angle 'θ'.
7. Is 'zero' a term of the Arithmetic Progression 31, 28, 25, ... ? Justify your answer.

SECTION – II

6 × 2 = 12

Note : (i) Answer **all** the following questions.
(ii) Each question carries **2** Marks.

8. Explain the terms in the formula $S_n = \frac{n}{2}[2a + (n-1)d]$.

9. Show that 2 and $-\frac{1}{3}$ are zeroes of the polynomial $3x^2 - 5x - 2$.

10. $A = \{x : x \in N$, and x is a factor of $30\}$; $B = \{x : x \in N$, and x is a prime factor of $30\}$. Draw Venn diagram for $A \cup B$.

11. If the measure of angles of a triangle are $x°$, $y°$ and $40°$, and difference between the two measures of angles $x°$ and $y°$ is $30°$, then find the values of $x°$ and $y°$.

12. If the distance between the two points (8, x) and (x, 8) is $2\sqrt{2}$ units, then find the value of 'x'.

13. Express 2016 as product of prime factors.

SECTION – III

4 × 4 = 16

Note : (i) Answer **all** the following questions.

(ii) In this section, every question has internal choice.

(iii) Answer **anyone** alternative.

(iv) Each question carries **4** Marks.

14. A) Use Euclid's division lemma, to show that the cube of any positive integer is of the form 3p or 3p + 1 or 3p + 2 for any integer 'p'.

OR

B) Prove that $\sqrt{3} - \sqrt{5}$ is an irrational number.

15. A) Draw the graph of the polynomial $p(x) = x^2 - 5x + 4$ on the graph paper. Find its zeroes from the graph.

OR

B) Draw the graph for the equations $2x - y - 4 = 0$ and $x + y + 1 = 0$ on the graph paper and check whether they are consistent or not.

16. A) On dividing $x^3 - 3x^2 + 5x - 7$ by $x^2 - 2x + 4$, if the remainder is in the form of $Ax + B$, find the values of A and B.

OR

B) $A = \{x : x \in N$, and x is a multiple of $4\}$;

$B = \{x : x \in N$, and x is a multiple of $6\}$;

$C = \{x : x \in N$, and x is a multiple of L.C.M. of 4 and $6\}$;

Find $A \cap B$. How can you relate the sets $A \cap B$ and C ?

17. A) If the points P(– 3, 9), Q(a, b) and R(4, –5) are collinear and $a + b = 1$, then find the values of a and b.

OR

B) The sum of the three terms which are in an Arithmetic Progression is 33. If the product of the first and the third terms exceeds the second term by 29, find the Arithmetic Progression.

9. Show that 2 and $-\frac{1}{3}$ are zeroes of the polynomial $3x^2 - 5x - 2$.

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SECTION – III

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OR

B) The sum of the three terms which are in an Arithmetic Progression is 33. If the product of the first and the third terms exceeds the second term by 29, find the Arithmetic Progression.

Time : 30 Mts.] **PART - B** [Max. Marks : 5

Instructions :

(i) Answer **all** the questions.

(ii) Each question carries ½ mark.

(iii) Answers are to be written in question paper only.

(iv) Marks will **not** be awarded in any case of over - writing, rewriting or erased answers.

I. Write the **CAPITAL LETTERS (A,B,C,D)** showing the correct answer for the following questions in the brackets provided against them. **10 × ½ = 5**

1. The logarithmic form of $a^b = c$ is ..... [ ]

(A) $\log_a c = b$ (B) $\log_b c = a$ (C) $\log_a b = c$ (D) $\log_b a = c$

2. If $3 \log (x + 3) = \log 27$, then the value of x is ..... [ ]

(A) 0 (B) 1 (C) 6 (D) 24

3. In the formula of n^{th} term of a Geometric Progression, $a_n = a \cdot r^{n-1}$, r denotes ..... [ ]

(A) first term (B) common ratio

(C) common difference (D) number of terms

4. Which one of the following rational numbers has terminating decimal expression ?

(A) $\frac{11}{7000}$ (B) $\frac{91}{21000}$ (C) $\frac{343}{2^3 \times 5^3 \times 7^3}$ (D) $\frac{21}{9000}$ [ ]

5. The common difference of an Arithmetic Progression in which $a_{25} - a_{12} = -52$ is ..... [ ]

(A) 4 (B) – 4 (C) – 3 (D) 3

6. Which one of the following statements is False ? [ ]

(A) Every set is subset of itself.

(B) Empty set is subset of every set.

(C) Intersection of two disjoint sets is empty set.

(D) Cardinal number of an infinite set is zero.

7. If the coordinates of the vertices of a rectangle are (0, 0), (4, 0), (4, 3) and (0, 3), then the length of its diagonal is ..... [ ]

(A) 4 (B) 5 (C) 7 (D) 3

8. The quadratic polynomial having $\frac{1}{3}$ and $\frac{1}{2}$ as its zeroes, is ..... [ ]

(A) $x^2 + \frac{5x+1}{6}$ (B) $-6x^2 - 5x + 1$ (C) $x^2 - \frac{5x-1}{6}$ (D) $6x^2 - 5x - 1$

9. Sum of 10 terms of the progression $\log 2 + \log 4 + \log 8 + \log 16 + ...$ is .... [ ]

(A) 45 log 2 (B) 90 log 2 (C) 10 log 2 (D) 55 log 2

10. Which term of the Arithmetic Progression 24, 21, 18, ... is the first negative term ?

(A) 8^{th} (B) 9^{th} (C) 10^{th} (D) 12^{th} [ ]

SOLUTIONS

PART - A

SECTION – I

1. If $p(x) = x^3 - 3x^2 + 2x - 3$ is a polynomial, then find the value of $p(1)$.

Sol. $p(x) = x^3 - 3x^2 + 2x - 3$

$p(1) = (1)^3 - 3(1)^2 + 2(1) - 3$

$= 1 - 3 + 2 - 3$

$= 3 - 6 = -3$

2. Find the value of k, if 2 is one of the roots of the quadratic equation $x^2 - kx + 6 = 0$.

Sol. $x^2 - kx + 6 = 0$

$(2)^2 - k(2) + 6 = 0$

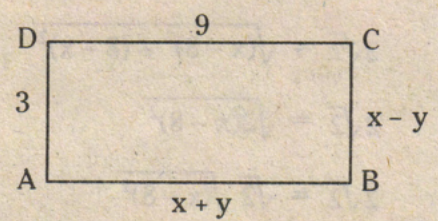
$4 + 6 - 2k = 0$

$2k = 10 \Rightarrow k = 5$

3. $A = \{x : x \in N$, x is a composite number and $x < 13\}$. Write set A in the roster form.

Sol. Roster form of $A = \{4, 6, 8, 9, 10, 12\}$.

4. In a rectangle ABCD, $AB = x + y$, $BC = x - y$, $CD = 9$ and $AD = 3$. Find the values of x and y.



Sol. $x + y = 9$

$x - y = 3$

($\because$ two pairs of opposite sides are equal, in a rectangle)

Solving the above equations, we get $x = 6$ and $y = 3$.

5. Write any two irrational numbers lying between 3 and 4.

Sol. $3 = \sqrt{9}$; $4 = \sqrt{16}$

Some irrational numbers lying between 3 and 4 are

$\sqrt{10}, \sqrt{11}, \sqrt{12}, \sqrt{13}, \sqrt{14}, \sqrt{15}$,

and 3.1121231234 .....

3.0101101110 .....

6. If the slope of the line passing through the two points (2, 5) and (5, 8) is represented by $\tan\theta$; (where $0° < \theta < 90°$) in trigonometry, then find angle 'θ'.

Sol. Points on the given line are (2, 5) and (5, 8)

$$\text{Slope} = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\tan\theta = \frac{8-5}{5-2} = \frac{3}{3} = 1 = \tan 45°$$

$\therefore \theta = 45°$ ($\because 0° < \theta < 90°$)

7. Is 'zero' a term of the Arithmetic Progression 31, 28, 25, ... ? Justify your answer.

Sol. $a = 31$; $d = 28 - 31 = -3$

$a_n = a + (n - 1)\,d$

$0 = 31 + (n - 1)(-3)$

$\Rightarrow n = \frac{34}{3}$

$\therefore$ zero is not a term of the given arithmetic progression.

Since 'n' is number of the term, it should be a whole number.

SECTION – II

8. Explain the terms in the formula

$$S_n = \frac{n}{2}[2a + (n-1)\,d].$$

Sol. $S_n = \frac{n}{2}[2a + (n-1)\,d]$

S_n = sum of 'n' terms of an A.P.

a = first term of A.P.

d = common difference of A.P.

n = number of terms

9. Show that 2 and $-\frac{1}{3}$ are zeroes of the polynomial $3x^2 - 5x - 2$.

Sol. $p(x) = 3x^2 - 5x - 2$

$p(2) = 3(2)^2 - 5(2) - 2$

$= 12 - 10 - 2$

$= 12 - 12 = 0$

$$p\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right)^2 - 5\left(-\frac{1}{3}\right) - 2$$

$$= \frac{1}{3} + \frac{5}{3} - 2$$

$= 2 - 2 = 0$

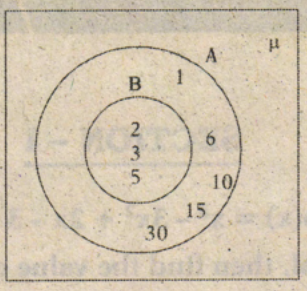
$\therefore$ 2 and $-\frac{1}{3}$ are zeros of p(x).

10. A = {x : x ∈ N, and x is a factor of 30} ; B = {x : x ∈ N, and x is a prime factor of 30}.

Draw Venn diagram for A ∪ B.

Sol. A = {1, 2, 3, 5, 6, 10, 15, 30}

B = {2, 3, 5}



11. If the measure of angles of a triangle are x°, y° and 40°, and difference between the two measures of angles x° and y° is 30°, then find the values of x° and y°.

Sol. $x + y + 40 = 180°$

($\because$ sum of angles of a triangle is 180°)

$x + y = 140$ — (1)

$x - y = 30$ — (2)

(1) and (2)

Solving the equations we obtain

$x = 85°$, $y = 55°$

12. If the distance between the two points (8, x) and (x, 8) is $2\sqrt{2}$ units, then find the value of 'x'.

Sol. Given points are (8, x) (x, 8) distance between them is $2\sqrt{2}$ units.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$2\sqrt{2} = \sqrt{(x-8)^2 + (8-x)^2}$$

$$2\sqrt{2} = \sqrt{2(x-8)^2}$$

$$2\sqrt{2} = \sqrt{2}\sqrt{(x-8)^2}$$

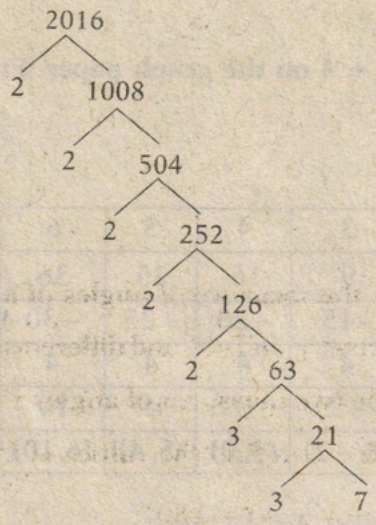
$$\pm 2 = x - 8$$

$x - 8 = 2$ or $x - 8 = -2$

$\Rightarrow x = 10$ or $x = 6$

13. Express 2016 as product of prime factors.

Sol.



$2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

$= 2^5 \times 3^2 \times 7$

SECTION – III

14.A) Use Euclid's division lemma, to show that the cube of any positive integer is of the form 3p or 3p + 1 or 3p + 2 for any integer 'p'.

Sol. Let 'a' be positive integer

$a = bq + r, 0 \le r < b$

$b = 3$ so $r = 0, 1, 2$

Then 'a' can be of the forms

$3q + 0, 3q + 1, 3q + 2$

Case (i) :

When $a = 3q$

$a^3 = (3q)^3 = 3(9q^3)$

$= 3p$ where $p = 9q^3$

Case (ii) :

When $a = 3q + 1$

$a^3 = (3q + 1)^3$

$= (3q)^3 + 3(3q)(1)(3q + 1) + (1)^3$

$= 3[9q^3 + 3q(3q + 1)] + 1$

$= 3p + 1$

where $p = 9q^3 + 3q(3q + 1)$

Case (iii) :

When $a = 3q + 2$

$a^3 = (3q + 2)^3$

$= (3q)^3 + 3(3q)(2)(3q + 2) + (2)^3$

$= 3[9q^3 + 6q(3q + 2)] + 2$

$= 3p + 2$

where $p = 9q^3 + 6q(3q + 2)$

So the cube of any positive integer is of the form 3p or 3p + 1 or 3p + 2 for any integer 'p'.

OR

B) Prove that $\sqrt{3} - \sqrt{5}$ is an irrational number.

Sol. Suppose $\sqrt{3} - \sqrt{5}$ is not an irrational number

$\sqrt{3} - \sqrt{5}$ is a rational number.

Let $\sqrt{3} - \sqrt{5} = \frac{p}{q}$

where $q \neq 0$ and $p, q \in Z$

squaring on both sides

$$3 + 5 - 2\sqrt{15} = \frac{p^2}{q^2}$$

$$\sqrt{15} = \frac{8q^2 - p^2}{2q^2}$$

$\therefore p, q \in Z$ & $q \neq 0$

$8q^2 - p^2$ & $2q^2 \in Z$ and also $2q^2 \neq 0$

So $\frac{8q^2 - p^2}{2q^2}$ is a rational number

but $\sqrt{15}$ is an irrational number.

An irrational number never become equal to a rational number.

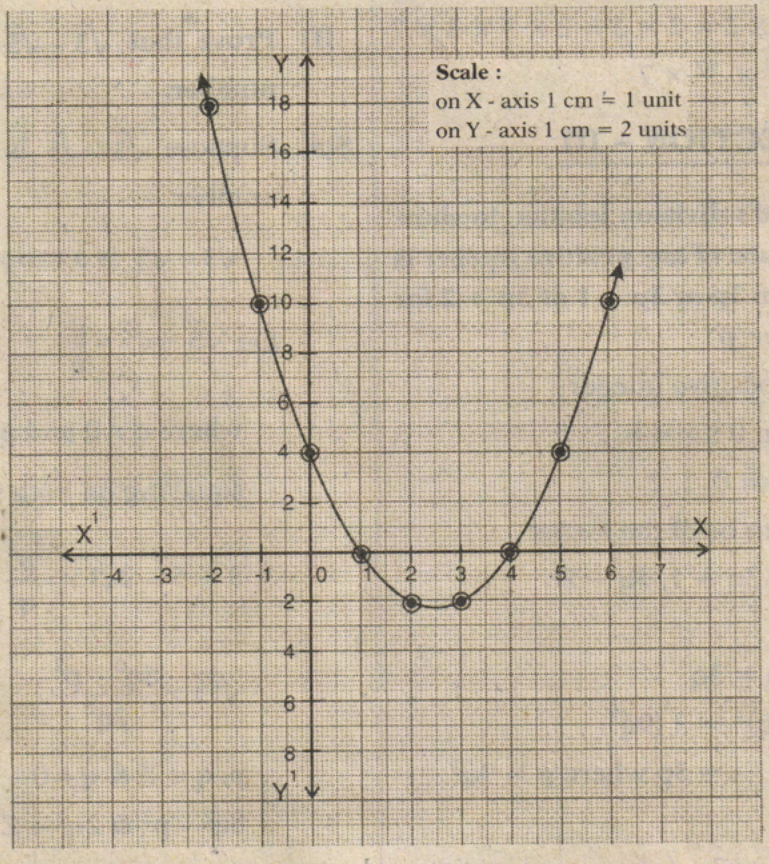
So our supposition that $\sqrt{3}-\sqrt{5}$ is not an irrational number is false.

$\therefore$ $\sqrt{3}-\sqrt{5}$ is an irrational number.

15.A) Draw the graph of the polynomial $p(x) = x^2 - 5x + 4$ on the graph paper. Find its zeroes from the graph.

Sol. $p(x) = x^2 - 5x + 4$

x	– 2	– 1	0	1	2	3	4	5	6
x^2	4	1	0	1	4	9	16	25	36
– 5x	10	5	0	– 5	– 10	– 15	– 20	– 25	– 30
4	4	4	4	4	4	4	4	4	4
y	18	10	4	0	– 2	– 2	0	4	10
(x, y)	(– 2, 18)	(– 1, 10)	(0, 4)	(1, 0)	(2, – 2)	(3, – 2)	(4, 0)	(5, 4)	(6, 10)



zeroes of the given polynomial are1 and 4.

OR

B) Draw the graph for the equations $2x - y - 4 = 0$ and $x + y + 1 = 0$ on the graph paper and check whether they are consistent or not.

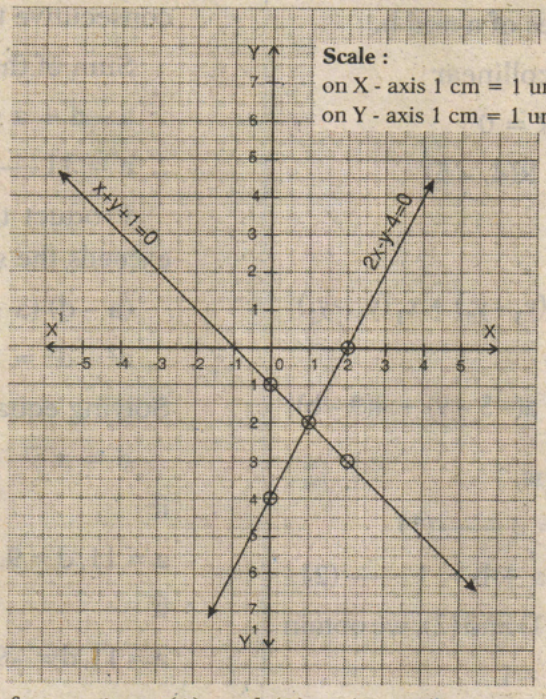
Sol. $2x - y - 4 = 0$ —— (1)

$y = 2x - 4$

x	0	1	2
2x	0	2	4
-4	-4	-4	-4
y	-4	-2	0
(x, y)	$(0, -4)$	$(1, -2)$	$(2, 0)$

$x + y + 1 = 0$ ..... (2) $y = -x - 1$

x	0	1	2
$-x$	0	-1	-2
-1	-1	-1	-1
y	-1	-2	-3
(x, y)	$(0, -1)$	$(1, -2)$	$(2, -3)$



Intersecting point of equations (1) and (2) is $(1, -2)$.

So $x = 1, y = -2$

$\therefore$ The given equations are consistent.

16.A) On dividing $x^3 - 3x^2 + 5x - 7$ by $x^2 - 2x + 4$, if the remainder is in the form of $Ax + B$, find the values of A and B.

Sol. $x^2 - 2x + 4) x^3 - 3x^2 + 5x - 7 (x - 1$

$x^3 - 2x^2 + 4x$

(–) (+) (–)

$-x^2 + x - 7$

$-x^2 + 2x - 4$

(+) (–) (+)

$-x - 3$

$(-1)x + (-3) = Ax + B$

$\therefore A = -1; \quad B = -3$

OR

B) A = {x : x ∈ N, and x is a multiple of 4} ; B = {x : x ∈ N, and x is a multiple of 6} ; C = {x : x ∈ N, and x is a multiple of L.C.M. of 4 and 6} ; Find A ∩ B. How can you relate the sets A ∩ B and C ?

Sol. A = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, ...}

$B = \{6, 12, 18, 24, 30, 36, 42, 48, ...\}$

$A \cap B = \{12, 24, 36, ...\}$

L.C.M. of 4 and 6 = 12

$C = \{12, 24, 36, ...\}$

$\therefore A \cap B = C$

17.A) If the points P(– 3, 9), Q(a, b) and R(4, –5) are collinear and a + b = 1, then find the values of a and b.

Sol. Points P, Q, R are collinear

$\Rightarrow$ area of $\Delta PQR = 0$

P(– 3, 9); Q(a, b); R(4, – 5)

Area of triangle

$$= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$0 = \frac{1}{2}|-3(b + 5) + a(-5 - 9) + 4(9 - b)|$$

after simplifications,

we get $2a + b = 3$ —— (1)

given equation $a + b = 1$ —— (2)

Solving equations (1) and (2), we obtain

$a = 2$ and $b = -1$

OR

B) The sum of the three terms which are in an Arithmetic Progression is 33. If the product of the first and the third terms exceeds the second term by 29, find the Arithmetic Progression.

Sol. Let $a - d, a, a + d$ be the three consecutive terms of an A.P.

Sum of three terms is 33

$a - d + a + a + d = 33$

$\therefore a = 11$ —— (1)

Product the first and third terms exceeds the second term by 29

$(a - d)(a + d) - 29 = a$

$a^2 - d^2 = 11 + 29 = 40$ —— (2)

Solving equations (1) and (2)

$d = \pm 9$

if

$a = 11, d = 9$, then the A.P. is 2, 11, 20, ...

if

$a = 11, d = -9$, then the A.P. is 20, 11, 2, ...

PART - B

1. A 2. A 3. B 4. C 5. B 6. D 7. B 8. C 9. D 10. C